

BETA RAY SPECTRA

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In Partial Fulfillment of the Requirements for the Degree
of Doctor of Philosophy

California Institute of Technology, Pasadena, California

1939

ABSTRACT

In part I the various methods of determining the energy distributions of electrons are described. Reasons are given for the choice of a semi-circular focusing spectrograph. The construction of the magnet, the spectrograph box, the field measuring devices and the coincidence Geiger-Muller counters are given in considerable detail.

In part II the experimental energy distributions of the $^{13}\text{Th C}$, ^{22}N , ^{32}Na , ^{13}P , and $^{22}\text{Ra E}$ nuclear electrons are described. In addition the distribution of the internally converted electrons from the ^{214}Pb 2.62 mev gamma ray and of the secondary electrons from the gamma rays of ^{13}N and ^{22}Na are given. The connection between these gamma rays and the complexity of the Beta spectra is discussed. In these cases where an independent estimate of the energy of the radio-active transformation is available there seems to be a discrepancy between this independent estimate and the experimental value for the end point of the continuous beta spectra. Several possible explanations of these discrepancies are discussed.

In the appendix the calculations of the distribution of the internally converted electrons from the ^{214}Pb gamma ray are given.

INTRODUCTION

The energy distribution of the electrons emitted from the nucleus of radio-active bodies has been studied for over twenty years. The general properties of this distribution have been discovered from the spectra of the natural radio-active bodies. Thus it is well established that the distribution is continuous (1), that there is an upper limit to the energy of the electrons emitted (2), and that the maximum number of electrons is emitted at an energy which is roughly $1/2$ or $1/3$ of the maximum energy. In addition it has been established through the natural radio-active bodies that the apparent maximum energy of the electron is the energy which is liberated in the nuclear transformation (3).

Along with the discovery of the artificial radio-activity and of positron emission came several new complications in the nature of these beta-ray spectra. These effects are also exhibited, in some cases, by the spectra of the natural radio-active bodies.

In the first place it appears that in many cases the spectra are not simple spectra, that is, instead of leaving only one nucleus they may leave several different structures often differing in several millionvolts energy. This energy is subsequently liberated either as a gamma ray or as discrete groups of β -rays from the atomic shells.

In the second place, it appears that the probability of beta-disintegration is not connected in a unique way with the maximum energy of the transformation and with the atomic num-

ber of the radio-active element, but that instead there seem to be certain more or less "forbidden" (4) transitions which take place.

In the third place, the energy at which the maximum number of electrons is emitted, or what we will call the peak of the distribution, is not a constant fraction of the maximum energy, but the relative position of peak and upper limit depends on the upper limit, the atomic number, the sign of the emitted particle, etc.

Lastly, the simple theory of beta decay of Fermi and Konopinsky and Uhlenbeck(5) leads to an expression for the shape of the beta ray spectra in terms of the atomic number and upper limit. The experimental shapes do not agree in detail with the theoretical shapes. The disagreement is not uniform, but is very minor in some cases and very aggravated in others.

Therefore, it has seemed advisable to systematically study the shapes of beta-spectra and to determine as far as possible how these shapes varied with:

- 1) Upper limit energy.
- 2) Sign of the emitted particle.
- 3) Atomic number of the emitting element.
- 4) Gamma rays associated with the beta ray decay.
- 5) The probability of the radio-active transition.

and to determine if there is a distribution which we could call simple, that is, a distribution which does not depend on accessory properties of the radio-active element, but in

a simple way on 1, 2, 3 above.

The beta radioactive transformations constitute only one type of nuclear reaction. The same element can often be made different ways, one involving beta decay and the others the emission of a proton or an alpha particle. Thus it is often possible to obtain an independent estimate of the total energy expended in a beta transformation from the masses of the nuclei involved. In some cases we should find the same excited states in the nuclei involved in beta transformations as are found with the heavy particle reactions.

Yet, although the determination of the endpoint of beta ray spectra is essential to the completeness of nuclear chemistry, we hope in addition that a study of the shape of these spectra will throw more light on the nuclear processes involved in radio-active transformations.

In this thesis we describe the apparatus for determining the beta ray distribution and give the results of a few preliminary experiments on the spectra of Th C-C', Na²², N¹³, Ra E, P³². The results that we have obtained do not answer any of the above questions definitely. In the case of Na²² and N¹³ the positron spectra are complex, and we have therefore begun an examination of the γ -rays of these two elements.

Part I APPARATUS

In measuring the energy of electrons, use is made of the fact that they are deflected by a magnetic field and that, when travelling perpendicular to the field, they travel in a circle of radius p such that $H\rho$ is proportional to the momentum of the electron.

$$H\rho = \frac{M}{e} = \frac{mc\beta}{e\sqrt{1-\beta^2}}$$

There are several ways of making use of this fact to determine the momentum:

- 1) Deflection and observation in a cloud chamber. (6)
- 2) A large scale electron microscope. (7)
- 3) Trochoidal focusing. (8)
- 4) Semi-circular focusing. (9)

We have adopted the last of these methods.

When the cloud chamber is used as a spectrograph as well as a detector, the cloud chamber is placed between two Helmholtz coils in such a way that the magnetic field is perpendicular to the plane of the chamber. The circular tracks are then photographed. This method suffers chiefly from two defects; since the electrons of all energies enter the chamber at once, it is impossible to examine carefully the upper end of the spectra. Any source which is strong enough to give an appreciable number of tracks at the upper end will give off thousands more tracks at a lower energy and thus fog the chamber. Secondly, since the electrons travel through

the gas of the chamber they may suffer many small angle deflections. These introduce errors into the curvature methods. The advantage of the cloud chamber is that it is possible to choose the tracks and discard those which have bounced or been deflected through a large angle.

In the electron microscope the electrons travel down a long tube. At each end of the tube is a coil which produces a lens shaped field inside the tube. The coil near the source makes the electrons of a given energy travel parallel down the tube and the lens at the other end focuses the electrons on a detector. A stop must be put along the axis of the tube. In some cases the tube is bent and the electrons are made to follow the bend by applying a homogeneous magnetic field perpendicular to the plane of the bend.

The disadvantages of this type of spectrograph is that it must have a very long tube in order to focus electrons of several million volts. Many small angle deflections can take place in the long tube, thus spoiling the intrinsic resolution of the apparatus. Its advantages are its intrinsic high resolving power and the large solid angle subtended by the source.

In the method of trochoidal focusing the electrons travel through an inhomogeneous field and are at the same time made to travel in a semi-circle. This method has very poor resolution and so could not be used to get an accurate determination of the shape of the beta ray spectra. It is however, very simple and convenient and makes use of a remarkably high solid angle.

In the method of semi-circular focusing the electrons are bent in a semi-circle from the source to the detector. The method does not really focus the electrons accurately, yet electrons which leave the source with an angular separation of about ten degrees, arrive at the detector fairly close together. The separation at the detector varies only as $(1 - \cos \theta) \propto \theta^2$, where θ is the angular separation of the electron paths at the source. This method suffers chiefly from this limitation on the solid angle. It also suffers from the fact that some electrons may be scattered into the detector. However, since high resolution can be obtained and high accuracy in measurements of Hp are possible the method seemed most desirable. Certain precautions can be taken against scattering. Many radio-active bodies can be made with sufficient intensity to overcome the limitation of the solid angle.

An electron detector must be used in conjunction with the above types of spectrographs. There are five types of instruments for detecting electrons,

- 1) The measurement of the charge transferred by the electrons directly. The Faraday cylinder works on this method.
- 2) Measurement of the ionization produced by electrons in an ionization chamber coupled with an electroscope or electrometer.
- 3) Detection of the electrons in a cloud chamber in conjunction with some other type of spectrograph.
- 4) Photographic plates.
- 5) The trigger action which the ionization of

an electron produces on the corona discharge of a gas condensor.

We obviously cannot make a choice of detector without some knowledge of the type of spectrograph and of the size and properties of the sources available. With the spectrograph which we have built the solid angle is about 1 in 500, while the width of the slits is such that the Hp spread that is measured is about 2 percent of the total Hp.

Let us consider how many electrons per minute are available from the following sources:

- 1) The internal conversion of the 2.6 mev gamma ray of Th. Pb. The sources available at the California Institute of Technology have an alpha particle activity of about 0.1 millicurie.
- 2) The bombardment of carbon with 10 μ a of 1 mev deuterons, producing radio-active nitrogen, 10 μ a of 1 mev deuterons are available from the pressure Van de Graph in the high voltage laboratory.
- 3) The radium E which can be collected from 1000 mc. of radon. This amount of radon is readily available from the radon plant in Kellogg since as much as 300 mc. of radon can be collected at a single pumping.

The following consideration will give the order of magnitude of the effects from the above sources.

TH. PB. INTERNAL CONVERSION.

Sources available .1 mc of α -particle activity

Number of Th Pb γ -rays per

α -particle $1/3$

Number of α -particles per

min. per mc. 2.3×10^9

Internal Conversion coefficient 1.8×10^{-3}

Solid angle 2.0×10^{-3}

Counts to be expected per minute

$$0.1 \times 2.3 \times 10^9 \times 1.8 \times 10^{-3} \times 2 \times 10^{-3} \times 1/3$$

$$= \underline{276 \text{ electrons per minute}}$$

POSITRONS FROM RADIO-NITROGEN.

Current of deuterons $10 \mu a$

Number of deuterons per minute $10 \times 60 \times 7 \times 10^{12}$

Approx. cross-section for

formation of N^{13} at 1 mev 10^{-25}

Approx. effective layer of

Carbon 1 cm air equivalent

Approx. effective layer of

Carbon contains $6.1 \times 10^{21} \times .0012/12 =$
 6.1×10^{17} carbon atoms.

Solid angle 2×10^{-3}

Approx. fraction of total

number of positrons col-

lected by slit at the

peak of the distribution .02

Counts to be expected per minute at the peak of distribution.

$$10 \times 60 \times 7 \times 10^{12} \times 6.1 \times 10^{17} \times 10^{-25} \times 2 \times 10^3 \times 2 \times 10^{-2} \\ = \underline{1000 \text{ positrons per minute}}$$

RECOIL ELECTRONS FROM A HYPOTHETICAL 1 MEV GAMMA RAY EMITTED
ONCE PER DISINTEGRATION BY THE ABOVE NITROGEN SOURCE.

Range of energies collected by slit

at 1 mev

20 kev

Loss of energy in Pb. for electrons

of 1 mev

1.1 mev/gr/cm²

Photoelectric absorbtion coefficient .19/gr/cm²

Counts to be expected

$$10 \times 60 \times 7 \times 10^{12} \times 10^{-25} \times 6.1 \times 10^{17} \times 2 \times 10^{-3} \times \\ .19 \times 2 \times 10^4 / 1.1 \times 10^6$$

$$= \underline{\overset{\text{photo-}}{85 \text{ recoil electrons per minute}}}$$

RADIUM E ELECTRONS.

After about two weeks 1000 mc. of radon, (half-life 3.7 days) will have decayed to Ra D (half-life of 2500 years) After ten days or so the number of Ra E disintegration, (half-life 4 days) is equal to the number of Ra D disintegrations. To calculate the number of Ra D disintegrations we must take the number of Rn disintegrations per minute and multiply this by the ratio $\tau_{\text{Rn}} / \tau_{\text{RaD}}$, where τ is the half-life. Thus there are $3.8 \times 10^7 \times 60 \times 10^3 \times 4 / 2.5 \times 10^3 \times 3.6 \times 10^2$ Ra E disintegrations/min. Again taking the solid angle of 2×10^{-3} and using the fact that only two percent of the total number of electrons are collected, there will be 760 counts/minute at the peak of the distribution.

Thus it seems that we will be able to get some sources of continuous beta rays which will give us from 300 to a few thousand electrons per minute at our detector, and that if these sources have gamma rays we will get from 50 to 500 recoil or internally converted electrons per minute.

Now if we look at the energy distributions that have been obtained by other observers we find that the intensity within 1 percent of the upper limit of the spectra is about 0.25 percent of the intensity of the peak, so that if we hope to measure the energy of the upper limit to 1 percent we will, with the sources available, have to be able to detect from one to ten electrons per minute.

Obviously any direct measurement of the charge collected is impractical. To obtain a microvolt with one electron the whole detector would have to have a capacity of 0.1 cm and even with ^{an} insulation resistance separating the plates of the detector of 10^{15} ohms there would be a leak of one electron per minute.

It is likewise impractical to detect the ionization of a single electron by means of an ionization chamber and amplifier. The limit of the sensitivity of such a detector is almost reached in the detection of protons which have an ionization from 50 to 500 times as large as that of the electron. The limitations on this method are again in part due to the small capacity of the ionization chamber required, but more fundamentally to the statistical fluctuation in the filament emission of the vacuum tubes and to the Johnson (10) effect of the high resistances necessary for the grid leak of the

first stage. These fluctuations produce a noise which is as great as that produced by the electrons in the ionization chamber.

Although an electroscope of the Lauritsen (11) type, which measures the ionization of electrons without vacuum tube amplification, has a very small capacity it takes some hundred of volts to separate the fibers. Therefore, the ionization produced by a few electrons causes a very small deflection which is comparable with the thermal and mechanical changes which are bound to take place with such an instrument. Even if these spurious effects could be eliminated the instrument would not be suitable for our purpose, since the time necessary to produce a measurable deflection of the fibre would exclude the use of radio-active sources with the same initial intensity which decay rapidly. When a radio-active substance is formed from a long lived parent substance its maximum activity is equal to the activity of the parent substance, but the total number of radio-active atoms present will be proportional to its half life.

The photographic plate affords a very convenient method of measuring electron intensities. It requires very little auxiliary apparatus and can be left without adjustment or care for very long times. Furthermore it is possible to record the intensity at several values of p , and thus of H_p , for the same field setting.

However, the intensity of the sources available prohibits the use of the photographic method for our work. To make a visible blackening, $10^6 - 10^7$ electrons per sq. cm. must

fall on the plate. Thus it would be completely impossible to examine weak sources.

There are normally $10^5 - 10^6$ grains developed without exposure. If these grains are removed, without fixing, and the plate redeveloped without any exposure, another $10^5 - 10^6$ grains are developed. Thus even a microscopic examination of the plate will be of no avail until it has been exposed to $10^5 - 10^6$ electrons. For high energies (greater than 2 million volts) the sensitivity of the photographic plate is even smaller because more electrons must then fall on the plate to make a grain developable.

Thus although some of the methods described are not without possible application to our problem, in every case they would mean stretching the sensitivity beyond the point which is practical and reliable.

Therefore, we have decided to use the trigger type of detector, a Geiger point or wire counter. In the Gieger-Muller (12) wire counter there is a cylindrical metal cathode and an axial wire with a potential difference of about 1000 volts between them. The counter is gas filled and the ionization produces a small and short corona discharge. With the proper counters and an electrical circuit this discharge stops in about 10^{-4} or less seconds. The discharge can be made to produce a large change of voltage on the grid of a vacuum tube. The details of the voltages and counters and amplifier will be described later. Here we want simply to point out the advantages and disadvantages of the method.

One disadvantage of the method is inherent in the fact that it detects the ionization rather than the electron itself. Therefore the counter chamber must contain some gas and must thus be separated from the spectrograph itself by a thin foil. Thus there is always a lower limit to the energy of the electrons which can be detected.

Secondly, although the efficiency of the counters is high, and can be made to be about 95 percent, this efficiency is to some extent a function of the voltage on the counters and the pressure of the gas in the counters. Therefore, these quantities must be kept reasonably constant. The degree to which the voltage must be kept constant depends on the counters. Some have a "plateau" in their voltage-sensitivity curve which is as great as 100 or more volts, while in others this plateau is negligible. Although there are some understandable rules for the construction of counters, they are famous for their erratic behaviour.

Thirdly, in Gieger counters, as in the instruments we have described, the cosmic radiation, local radio-activity and the γ -rays from the source are detected as well as the effect we are measuring. Since the "background" is subject to statistical fluctuations, it is difficult to measure effects which are orders of magnitude smaller. The "background" varies considerably, but is about 1 count per minute per cc. of volume of the counter. Since it is difficult to make extremely small counters work reliably, the backgrounds are usually 15 to 30 counts per minute. Thus for a background of 20 per minute we must take a 35 minute run, to be 80 percent certain

that a count of 21 per minute is not a statistical fluctuation.

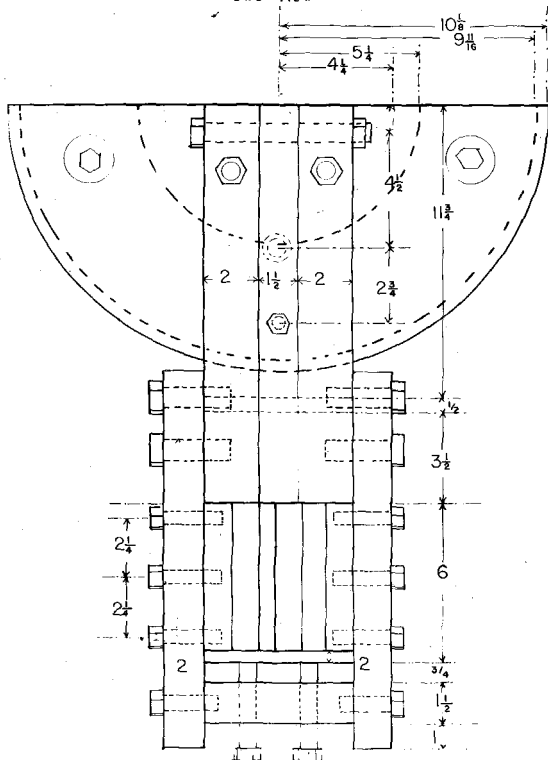
However, this background can be considerably reduced if we measure only those electrons which are travelling in given direction. This directional sensitivity can be obtained by measuring only the coincidences between two or more counters. The definition of the direction and thus the lowering of the background is greatest when the two coincidence counters are far apart. However, since the electrons emerging from the spectrograph are emerging with an angular separation of about ten degrees, it is not practical to define the direction more precisely than ten degrees. Thus for our purpose the use of coincidences reduces the background to about 4 per minute. With this background we can be 80 percent certain that a count of 5 per minute is real after 6 minutes.

This discussion may give a somewhat misleading impression of the ease with which accurate results can be obtained. It should be pointed out that "80 percent certain" is not very certain, and that any statistical discussion presupposes the constancy of all non-random effects, such as the value of the magnetic field and the sensitivity of the counters.

These more or less qualitative considerations determined our choice of spectrograph and detector to semi-circular focusing and Geiger-Muller counters respectively.

SPECTRAPH MAGNET

FIG 1
END VIEW



APPARATUS

Construction

The Spectrograph consists of

- 1) An electro-magnet to deflect the electrons.
- 2) An evacuated chamber or box through which the electrons travel.
- 3) Geiger-Muller coincidence counters.

The Magnet

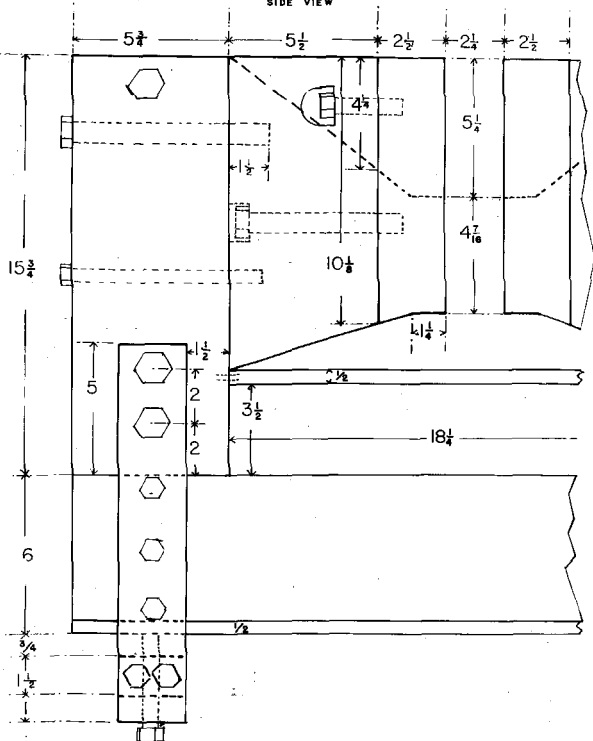
The dimensions and the shape of the magnet are shown in Figure 1. The maximum field obtainable with the $2\frac{1}{4}$ inch gap is about 2200 gauss, / With a radius of curvature of about 19 cm. we can focus electrons of 13 and 18.5 mev with the large and small gaps respectively.

As the dispersion of a spectrograph depends solely on the product $H\rho$, and the resolution on the arrangement of slits and on the source size, these factors do not enter into the considerations of the size of the magnet. Two factors influence us to reduce this size, 1) the difficulty of producing enough flux to maintain a uniform field over a large area, and 2) the linear reduction of the solid angle with the radius of curvature. The fact that this reduction is linear is due to the focusing properties of the spectrograph.

On the other hand, it is possible to shield the counters from the gamma rays given off by the source only if the spectrograph is sufficiently large. With a source that gives about 70 recoil electrons per minute, if we assume an absorption coefficient of 0.4 cm^{-1} , we would get ten quanta per minute at a distance of thirty cm. through thirty cm. of Pb.

FIG 1
SIDE VIEW

SIDE VIEW



In the second place, with a long path length, it is more likely that electrons which are scattered by the slits near the source will finally be deflected so as not to reach the counter.

It is, however, primarily in order to be able to remove the source from the counters that we have sacrificed the solid angle and have made a fairly large spectrograph with a path of 19 cm. Not only the gamma rays actually produced by the source, but in the case of continuous bombardment, the X-rays and electrical disturbances at the target can thus more easily be kept from affecting the counters.

The area over which the field must be uniform can be materially reduced by limiting the pole pieces to the paths that the electrons will follow. Dr. E. Lyman (13) at the University of California, had made such a magnet and had found that the field was sufficiently uniform. The pole pieces of our spectrograph are pieces of Armco electrolytic iron 2 inches thick and bounded by concentric semi-circles of 23 and 15 cm. radius. The Armco iron was used for the pole faces because its extreme purity eliminates the possibility of the local variations of the magnetic permeability. The rest of the magnet is made of mild steel.

The area of the pole faces is about 680 cm^2 , while the area of the core is only about 200 cm^2 . It is important that this difference be large for the shape of the lines of force near the edges of the gap, and thus the edge correction to the magnetic field will change with the field if the pole pieces are allowed to approach saturation. The shape of the

lines of force near the edges of the pole pieces will change only slightly with the field, until, near saturation the permeability approaches unity. The fact that the core of the magnet limits the flux and thus prevents the saturation of the pole pieces guarantees the proportionality of H_p with the H that is measured at only one point of the field.

In order that the field be uniform along the path of the electrons the pole pieces must be strictly parallel. When they are made parallel to within a mil, we have verified that the field along the 19 cm. radius varies by less than 1 part in 500 to within 3 cm. of the ends.

The field varies by 3 percent or 4 percent over the width of the pole pieces, that is, from the inside to the outside radius. However, here again, the field is constant to within 1 part in 500 over a central region of 4 cm. Therefore, in designing the box, the path of the electrons was kept within these four cm.

This is shown in fig. 5.

The END CORRECTIONS due to the fact that the field drops off at both ends of the electron path, that is, at the source and at the counter, cannot be neglected. However, Hartree (14) has shown that by using the integral $\bar{H} = \frac{1}{L} \int_0^L H \cdot dl$ we obtain an average field for the whole path. The non-uniformity of about 1 percent is only over about 6 percent of the path length so that the Hartree correction will be small if not negligible. However, as mentioned before, the change in shape of the lines of force at the edges will not be very great, therefore the variation in the Hartree correction

at different fields will be altogether negligible. We have not used the Hartree integral to calculate the field, but simply have used its existence to justify our procedure of calibrating the H_p of the spectrograph by measuring H at only one point in the field. We have used the internally converted electron of the Th. Pb gamma ray whose H_p is 10,000 to effect this calibration.

The following table shows some of the important features of the magnet.

Table 1

Mild Steel (Properties)

B_{\max}	18,000 gaus
$H_{B_{\max}}$	150 gaus
B_{\max}	120

Magnet

Area of cross section of core and columns	200 cm^2
Area of pole faces	680 cm^2
Flux at B_{\max} = 18,000 x 200	3.6×10^6 gaus cm^2
Field in gap at B_{\max} (no stray field)	5,300 gaus
Magnetic resistance of core and columns	5.4×10^{-3}
Magnetic resistance of $2\frac{1}{4}$ " gap	7.4×10^{-3}
Total resistance, R	1.28×10^{-2}

fig 5

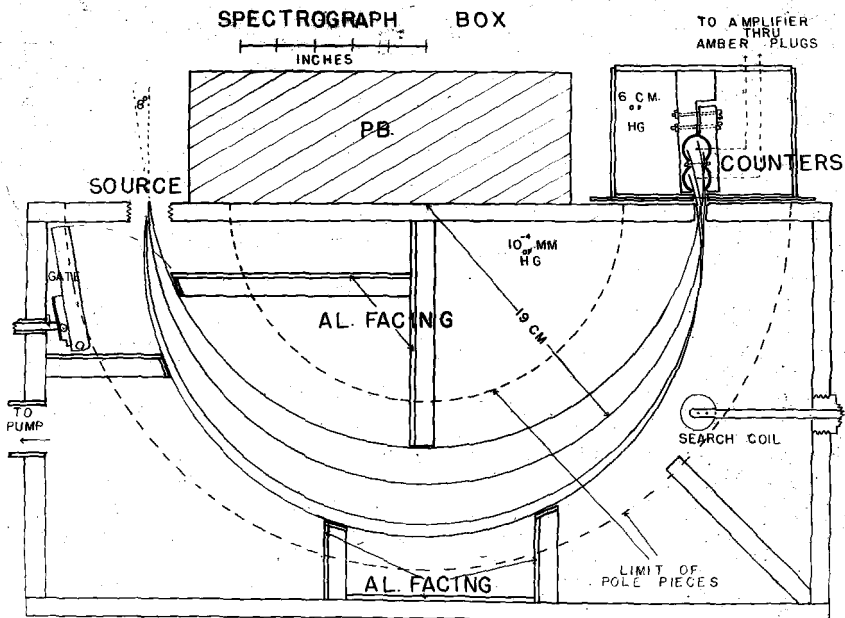


Table 1 cont.

Coil

Number of turns	1350
Resistance	1.83 ohms
Maximum current without overheating	30 amps
$4\pi n i = 1/10 \times 30 \times 1350 \times 4$	4.85×10^4
Flux at maximum current	3.8×10^6 gauss cm ²

The Evacuated Box

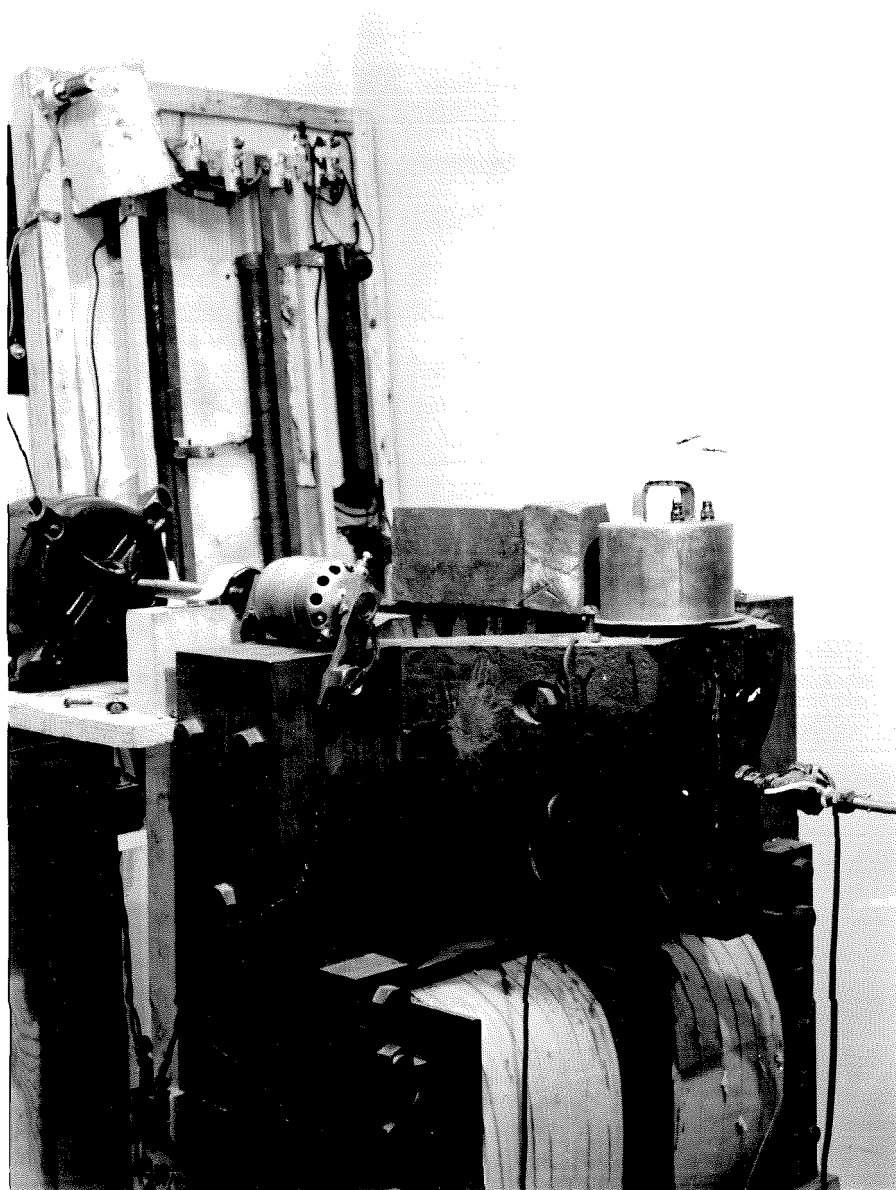
Fig. 5 gives the details of the box for the spectrograph that we are using.

Its essential features are:

- 1) An entrance for the electrons at the source and an exit slit for the electron at the counter.
- 2) An essential defining slit near the source and a few baffles to help keep the scattered electrons from reaching the counters.
- 3) Aluminum facing to reduce the scattering of the electrons which hit the edges of the baffles and the sides of the box.
- 4) A means of evacuating the box.
- 5) A means of measuring the field near the path of the electrons.
- 6) A gate to keep the emitted electrons from reaching the source when necessary.

The source holder is made of aluminum. The inner

Fig. 6



aluminum ring can be screwed so as to clamp a cellophane window. The source may be placed either above or below the cellophane window. Since this window slows the electrons down and, at low energies of 100 or so Kev, actually absorbs some of them, it is best to put the source in the vacuum under the cellophane. However, for short life sources this is impractical. During the time taken to pump down the box, the source will decay.

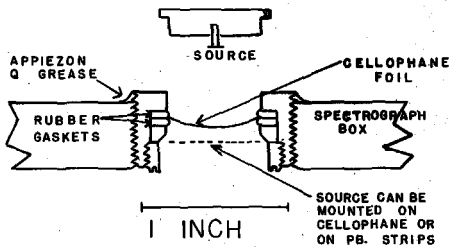
The slit pieces and baffles are thicker than necessary for most of the work that we have done. However, they were made thick with the view of examining 10 to 15 mev spectra. The slit pieces 1 and 2 are arranged so that no direct particles can get into the body of the box. They have been spaced so as to limit the distribution which would be due to a monochromatic line to the width of the slit at the counters. This slit is 2 percent of the radius of curvature.

The usefulness of the remaining baffles is somewhat doubtful. All the necessary definition can be obtained with the first slit and the counterslit. However, since there can be so many electrons bouncing around the source-half of the box, especially during continuous bombardment of a target with an A.C. tube, we have put some baffles in the source-half. We have left them out of the counter-half as they probably do as much harm by scattering electrons into the counters as they do good by preventing scattered ones from entering.

Henderson (15) has tried a very elaborate system of such baffles and reports little difference in performance with and without them.

AL. SOURCE HOLDER

FIG. 8



The aluminum facing is 1/16 inch thick and is screwed on to all the brass surfaces which are exposed to the electrons. The baffles and ^{slits} are mounted on aluminum side pieces and can be taken out of the box as a unit.

The box is evacuated through a large tube on one end by a Hyvac pump. When the box is tight this pump brings the pressure down to 10^{-4} mm. of Hg. At this pressure electrons make on the average less than one collision with an air atom (radius 10^{-8} cm.) during the entire path length. The chance of any appreciable scattering of a 100 Kev electron by the gas is therefore negligible, the nuclear radius being about 10^{-13} cm.

To measure the pressure we use a small discharge tube and Ford spark coil. We have found, by using a McLeod gauge, that when the vacuum becomes "hard" for this discharge tube and coil, the pressure is about 10^{-4} mm. of Hg. We do not need to know the pressure accurately, but only require that it be low enough.

One side of the brass box is soldered to the edges of the box and the other side is screwed and waxed. The solder, however, breaks, for although the baffles inside give some support to the sides, the bending upon evacuation is sufficient to crack the solder. Therefore both sides of the box have to be treated with wax and shellac. Some skiwaxes, consisting of shellac and beeswax have been found better than ordinary shellac. Although the box can be made tight, it is not very well designed for this purpose. The screw holes are not separated from the vacuum, so not only do the screw

heads have to be waxed or soldered but the holes under the screws fill up with air if the box is left open for any length of time. It then takes a correspondingly long time to pump the box out again. Some sort of tongue and groove joint with the groove inside the screws would have proved more convenient.

We have included a gate in the box but have seldom used it, having always found it more convenient to reduce, increase or reverse the field. Part of our hesitation to use it is due to the fact that it is badly placed. When the source emits gamma rays these rays can eject recoil electrons from the back side of the closed gate and these electrons could then enter the counters.

Field Measurements

We have not made any absolute measurements of the field or the radius of curvature, p , but instead have calibrated the spectrograph with electrons of known energy. We have used the electrons of $H_p = 10,000$ emitted by the internally converted 2.62 mev gamma ray of Th. Pb. Since p is always the same with our apparatus, the ratio between H_p and any linear measure of H which we find convenient will be a constant.

We have mounted inside the box a small search coil which is as near as possible to the path of the electrons. We cannot put it in the path since it would scatter the electrons. This search coil is free to rotate through 180° , and is connected with a long rod to another similar search coil which turns inside a long solenoid. The two coils are connected in

series with each other and a galvanometer. The current through the solenoid is adjusted in magnitude and direction until there is no deflection of the galvanometer when the coils are rotated through 180° . The current which satisfies this adjustment is what we have taken as the linear measure of our field. The field inside the solenoid varies linearly with the current through it, and the e.m.f. developed by the search coils is a linear function of the fields they turn in. There is a variable shunting resistance across the galvanometer which determines its sensitivity and proves convenient in finding the magnitude of the field and in protecting the galvanometer when the field is turned on and off.

If the shape of the search coils and the fields through which they turn were identical the galvanometer mirror would not move at a true balance. We were able to obtain this condition only approximately, with the result that the galvanometer, even at true balance, deflects first to one side and then to the other finally coming to its original position. Therefore, although in principle the accuracy of this method of determining relative field strengths is limited only by the accuracy to which the current through the solenoid is read, in our case the accuracy is not better than 0.5 percent. If the time during which the search coils are rotated were extremely short compared to the period of the galvanometer, the inaccuracy would be very much reduced. In our case, however, the search coil in the box is rotated through the packing gland of a steam valve so that it is difficult to

"snap" it around.

Figs. 5 and 6 illustrate our field measuring devices.

To obtain a direct reading measure of the field, we have taken the field coils out of a small aluminum-cased D.C. motor. This field-less motor is placed in the field of the magnet and rotated by a 220 v, 1/3 h.p. synchronous motor. The e.m.f. generated by the fieldless motor is then measured on a D.C. voltmeter which is in series with 2,000 ohms. The voltages vary from 1 to 3.5 volts. We have calibrated this device against the search coil and strangely enough, that is despite the iron armature of the motor, the voltage it develops is proportional to the current through the solenoid. Unfortunately, the proportionality factor varies as much as 3 percent with the past history of the magnetic field. We were not able to place this motor generator in between the pole pieces because it is too bulky. Instead we have put it in the stray field on the top of the magnet. This stray field although in general proportional to the field in the box probably depends somewhat on the part of the hysteresis cycle which the magnet has gone through.

This variation does not materially limit the accuracy or convenience of this direct reading flux meter. It merely necessitates a single calibration against the search coil after each radical change in the field.

The error in the energy determinations arises from two inaccuracies.

- 1) The inaccuracy in the field measurements amounting to not more than 0.5 percent.
- 2) The difficulty of determining the position

of a line or bzw. the end of a β -ray spectrum.

The second inaccuracy varies from experiment to experiment, depending on the care with which the head or experimental limit is measured.

We can base our calibration on either the head or peak of the line from the Th. internally converted electrons. The two different calibrations would give the value for the minimum value of p. or the value of p. at the middle of the slit respectively. The peak usually falls 1.5 to 2 percent behind the head of the line, depending on the source widths, (see Fig. 16). When we wish to get the energy of a line from its peak we will have to use the fact that the peak of the internally converted line, Hp 10,000, is at the field which is balanced by 0.982 amps through the solenoid. (Figs. 16 and 17).

We have used the head of the internally converted line (0.99 amps) for our calibration since the position of the peak of the photo-line is complicated by the fact that electrons are generated in a finite thickness of lead. This "head" calibration gives a value to the end point of continuous beta-ray spectra which may be an under estimation of the energy by about 0.5 percent. Thus the errors in our energy measurement will at most be 1 percent. This error, of course, has nothing to do with the question of scattered electrons. It concerns only the mechanical acts of reading meters and of extrapolating graphs.

One check on the accuracy of our field measurements is afforded by the energy of the recoiled electron from the

annihilation radiation of the positron. The energy of the radiation is 0.5107 mev. Subtracting the K-ionization potent of Pb of 90 kev, we get for the expected energy of the recoil electron 0.421 mev. The observed value is 0.420. This agreement is well within the experimental error.

The Counters

A geiger-Muller counter consists of a cylindrical cathode and a concentric wire anode. The action of such a counter is probably as follows: when an ionizing particle enters the counter, the positive ions are drawn to the wall and the electrons are accelerated to the wire. These accelerated electrons produce further ionization and also excite some of the atoms. There results therefore, a small avalanche of electrons to the wire. If this were all that occurred the charge coming to the wire would be proportional to the ionization of the incoming particle. If the potential difference is not sufficiently great, this is effectively all that happens. Counters in this condition are called proportional counters and are frequently used to detect the ionization of heavy particles. The ionization collected, although proportional to the primary ionization, is often many times greater, so that the counters are also called multiplication counters.

However, the atoms which are excited by the accelerated electrons give off radiation and this radiation ejects fresh electrons from the walls. If the potential difference is sufficiently great these photoelectrons will start a fresh avalanche. Then the number of subsequent avalanches and thus

the charge on the wire will have no simple relation to the ionization of the impinging particle. This is the action of the trigger type counter. The important thing in this case is that the corona discharge which has been set off does not continue forever. To prevent this from happening three conditions can be fulfilled. 1) The chance of knocking a photoelectron out of the wall can be reduced, 2) the potential difference between the cathode and anode can be reduced when the discharge starts, and 3) meta-stable states in the gas, which might give off a quantum and start a new discharge after the original one had been quenched, can be eliminated. The first is determined by the nature of the walls, the second by the circuit, and the third by the gas.

Many different wall materials and coatings have been tried. These include noble metal walls, (16) and oxide and lacquer coatings. Certain types of oxide coating seem to be the best. However, they require baking out and reducing with hydrogen. They are therefore only practical with counters that can be sealed in glass. We have found that a sulphide coating on copper is quite satisfactory. The coating is made by first cleaning the copper in concentrated nitric acid and then dipping the counters in a solution of ammonium sulphide.

Many different gases and combinations of gases have been tried: air, argon, helium and hydrogen. It has been found however, that the meta-stable states, and thus repeated or fuzzy discharges, can best be eliminated by certain combinations of gases. Presumably collisions of the second kind between these particular combinations of gases are responsible

for the elimination of the meta-stable states. Such combinations are argon and helium, argon and air, air and alcohol. I have been told that the combination of argon and alcohol does not work.

We use, because of its simplicity, 2 cm. mercury of alcohol vapor and 4 cm mercury of air.

The shape of the counters undoubtedly influences their performance. We have for instance never been able to get counters with plane cathodes to work satisfactorily. (On the other hand Mr. Tomlinson calculated and made a pair of counters whose walls although they had slits in them, formed surfaces of equal charge density around the wires. These counters are made of brass which does not coat well. They give a poorer performance than less ideally shaped ones with a proper coating.)

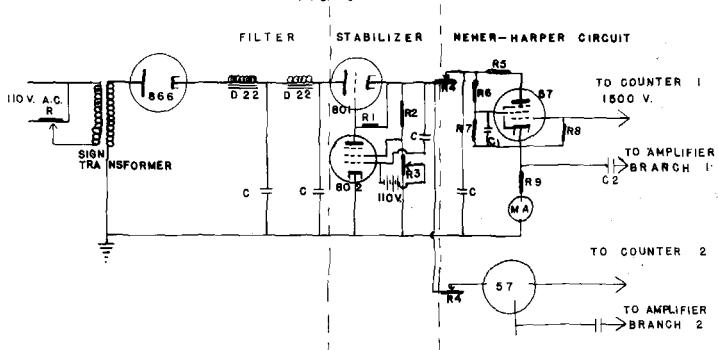
Our cylindrical counters with longitudinal slits in them do give adequate performance. They have a "plateau" of about 100 volts. It would be possible to make better counters by making them of thin copper foil cylinders. But in order that ^{an} electron go through both counters, and thus register a coincidence, it would have to go through three thicknesses of copper foil. It would thus be impossible to measure low energy electrons.

We therefore have put the two counters in the same box, separated only by a very thin cellophane or aluminum foil. This foil prevents the ultra-violet light from the discharge of the one counter from entering the other. The box with a pressure of 6 cm. mercury inside is then separated from

R_1	0.05 meg
R_2	2 meg
R_3	0.5 meg
R_4	1 meg
R_5	0.3 meg
R_6	5 meg
R_7	1 meg
R_8	10 meg
R_9	4 meg

C	2 μf
C_1	0.0001 μf
C_2	0.1 μf

FIG. 9



the vacuum in the spectrograph by a cellophane foil which has only about 45 kev stopping power for electrons. We have found that cellophane is slightly porous, the porosity can be eliminated by coating the cellophane with a little Canada balsam dissolved in ether.

The foil is waxed in between two brass discs which are screwed together and the whole counter box is sealed to a plate on the spectrograph box with Apiezon Q grease.

The Voltage Supply to the Counters.

Fig. 9 shows the circuit of this voltage supply. It consists of three units.

- 1) A transformer, rectifier and filter.
- 2) A stabilizer to maintain constant voltage on the counters when the A.C. input to the transformer varies.
- 3) A circuit which lowers the voltage on the counters when the discharge begins.

With about 70 volts on the primary of the transformer the filter delivers about 2000 volts to the stabilizer. The stabilizer is set to give about 500 volts drop across the triode. This voltage is then supplied through a 57 vacuum tube and 10 meg ohms to the wire of the counters.

The action of the stabilizer is quite simple. The plate current of the pentode, 802, is taken from the filtered voltage through a high resistance. The plate of this pentode is connected to the grid of the triode so that the current through the pentode, and thus through its high load resistance

determines the potential of the triode grid.

The current through the pentode is then determined by the potential on the grid of the pentode. This potential is derived from a voltage divider on the output voltage of the stabilizer, from which a constant battery voltage is subtracted.

Thus a small rise in the current through the voltage divider is amplified by the pentode to make a larger voltage drop across the triode.

With some types of stabilizers (17), an increase in the filtered voltage can produce either a rise or fall in the stabilized voltage, depending on the adjustment of the stabilizer. The region over which neither a rise nor fall is produced, that is, where the stabilizer functions, varies with the input and output voltages. Thus for a change in the output voltage two adjustments must be made.

With our type of stabilizer a rise in the filtered voltage always produces a small rise in the output voltage. But this rise depends very little on the input and output voltages and is determined chiefly by the amplification factor of the pentode.

With the first type of stabilizer mentioned there is one voltage for a given adjustment for which dE_o/dE_f is zero. Immediately above and below this voltage, dE_o/dE_f will be small and positive above and negative below, where E_o = the output voltage, and E_f = the voltage delivered by the filter circuit. dE_o/dE_f becomes larger, the further away

it is from the balance voltage at which the stabilizer is operating. For our type dE_0/dE_f is never zero but is always small and positive.

A change in the primary voltage of 10 volts, produces a change in filtered voltage of about 200 volts and in the stabilized voltage a change of less than 5 volts.

The third part of the voltage supply is the modified Neher-Harper (18) circuit in which the walls of the counters are grounded. The Neher-Harper circuits greatly reduce the time constants of the counter discharge in comparison with the conventional high-resistance capacity type of quenching circuits. These latter circuits involve resistances of 10^{10} ohms and capacities of 10 or more cm.

We have used the modified form of the Neher-Harper circuit for two reasons: since the counter can be grounded it is possible to put it very close to the exit slit of the box, thus collecting all the emergent electrons with a comparatively small counter. In addition the circuit requires only one lead from the voltage supply and amplifier to the counters and thus reduces the difficulties of simultaneous insulation, shielding and maintaining a low capacity for the leads.

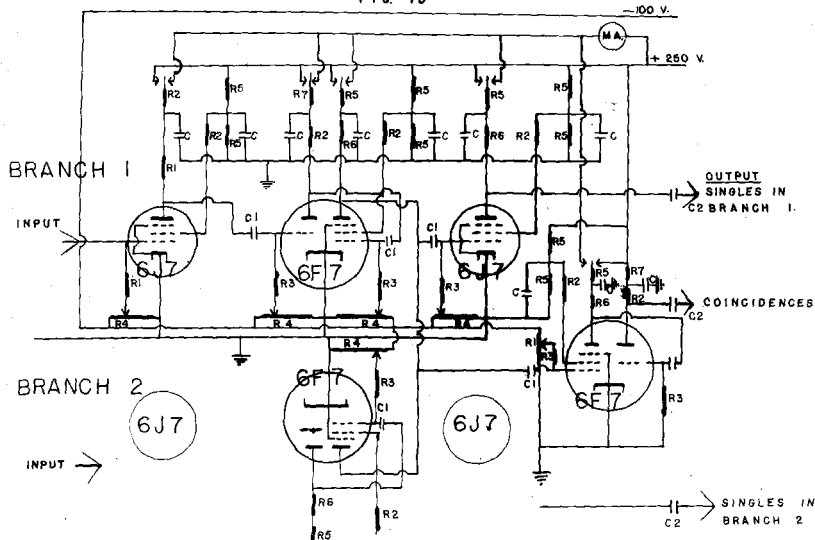
The action of the circuit, Fig. 9, is as follows. The high voltage is supplied to the counters through a 0.3 megohm resistor, a 57 vacuum tube and a 10 megohm resistor. The tube is part of a voltage divider which consists of the 0.3 megohm, the tube and 4 megohms (through a milliammeter) to ground. Ordinarily most of the voltage drop occurs

R_1	1 meg
R_2	0.1 meg
R_3	0.3 meg
R_4	0.1 meg variable.
R_5	0.05 meg
R_6	0.5 meg
R_7	0.01 meg

C	8 μf
C_1	0.001 μf
C_2	0.005 μf

AMPLIFIER FOR COINCIDENCES OF GEIGER-MÜLLER COUNTERS.

FIG. 10



push button switch in each plate circuit.

The three pentodes would certainly provide ample amplification to work the recording thyratrons. The triodes do not add much to the gain of the amplifier but reverse the sign of the impulses on the Rossi coincidence circuit and on the thyratrons, a negative impulse being required in each case.

There are some precautions in building such an amplifier which it might be well to set down.

- 1) The two branches of the amplifier must be carefully shielded from each other and from the output. The need for this precaution is to prevent feed-back and make it impossible for a strong kick in one branch of the amplifier to produce a kick in the other branch, thus registering a spurious coincidence.
- 2) The kicks must be amplified until they are strong and as nearly as possible all of the same size, before they act on the two Rossi coincidence tubes.

The action of the Rossi circuit is best when both tubes are completely quenched. If the tubes are always quenched by the kicks, the thyatron bias can be set permanently and does not have to be varied for different counters. The impulses of the counters may vary for a number of reasons. The size of the kick depends not only on the capacity and thus on the dimensions of the counters, but also on the voltage on the counters, or rather the amount this voltage is raised above

the threshold for trigger action. During fast counting the voltage on the counter may not always return to its maximum value before another ionizing particle enters the counter. Therefore, for some of the kicks the voltage above the threshold will be smaller and the kicks correspondingly small. Of course for extremely fast counting some of the electrons may enter before the voltage is risen to the threshold and thus not be counted at all. This we will discuss later.

We have provided three output terminals to the amplifier so that we can, if desirable, record simultaneously the singles and coincidences of the two counters. It is occasionally desirable to make such a simultaneous recording in order to test the efficiency and performance of the Gieger counters.

Due to geometry, that is, to the fact that we have not made the top counter large enough to admit all the electrons through its slit, the top counter registers only one-half as many electrons as the bottom counter. If E_1 and E_2 are the number of electrons in the top and bottom counter minus their common cosmic ray background E_1/E_2 is $\frac{1}{2}$.

On the other hand the efficiency of the counters, is high. Almost all the electrons which go through them are counted. If N is the number of coincidences then the efficiency is $(N/E)^2$ where E is the number of electrons which go through both counters. For our counters the efficiency is about 95 percent.

THE RECORDER.

We have used a more or less standard (19) circuit for a scale of eight recorder. The Cenco counter moves one division for every eight impulses and the ^{three} neon lights interpolate between 0 and 7. To interpolate one simply adds the numbers (1,2,4) under those neon lights which are lighted.

COUNTING LOSSES.

When too many electrons are entering the Gieger counters per minute some of these electrons are not registered on the recorder. These counting losses can arise in two ways.

- 1) If two impulses arrive in very rapid succession the second one is simply ignored and produces no effect on any part of the recording devices.
- 2) The second impulse is not counted, but is also not ignored. It lengthens the recovery time of some part of the recording system.

If we have the first type of losses, then when we send more and more electrons into the counter per minute, the number recorded will at first increase, then reach a broad maximum and finally decrease. If, on the other hand, our losses are of the second type, the number of particles recorded as the number of electrons increases, will also increase until blocking sets in. Then no particles will be recorded.

Geiger counters when actuated by a Neher-Harper circuit and Thyratrons are of the first type. The second impulse

COUNTING LOSSES

FIG 12

$\frac{N_0}{N}$

5

4

3

2

1

QUENCHING

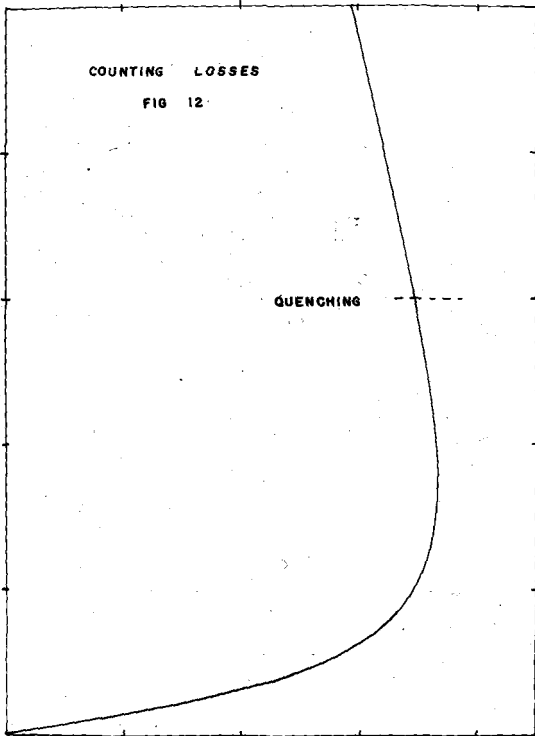
$\frac{N_1}{N}$

.1

.2

.3

.4



is completely ignored.

Vacuum tube amplifiers and mechanical recorders are of the second type. The second impulse makes them insensitive for a protracted time.

Since we are using a Neher-Harper circuit and since the mechanical recorder is slowed down by the scale of eight, our losses will be of the first kind if the time constants of the vacuum tubes are short compared with those of the thyratrons. Such is the case.

L. I. Schiff (19) has given a simple method of correcting for losses with recorders of the first type. Using his method, we have constructed a curve giving our losses as a function of the counting rate. Fig. 12.

From this curve it can be seen that when counting at the rate of 1,000 per minute we fail to record about 5 percent of the electrons entering the counter. In practice we have not counted more than 500 per minute.

For high counting rates Schiff's method is probably not applicable to our amplifier since the time constants of the amplifier are not negligibly short compared with those of the thyatron recorder. This can be seen experimentally by bringing a strong source of gamma rays close to the counter. The recorded counts at first increase, come to a maximum number per minute, and then decrease, showing that type one losses are predominant. However, when the source is moved still closer to the counters, the counts suddenly cease, showing that type two losses are also occurring. When the output of the amplifier is connected to ear phones instead of to the thyatron recorder, the counts per minute seem to

steadily increase, without passing through a maximum, as the source is brought nearer the counter. Finally they stop altogether, showing that the quenching and the type two losses occur in the amplifier, not in the recorder, as is to be expected.

PART II

BETA-RAY SPECTRA

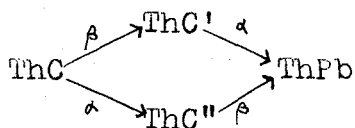
Experimental distributions.

In this part we will present the results of our investigations of the spectra of several radio-active elements. We have re-examined with some care the end point of the Th C-C', P^{32} and Ra E distributions. In addition we will give the data that we have thus far obtained on the spectra of radio-Na²² and radio-N¹³. This data is still incomplete but throws considerable light on the nature of these two spectra.

Th C-C'

The spectrum of the thorium bodies has been examined by Gurney (20) and more recently by Henderson (21). The energy and the intensity of the group of internally converted electrons of the 2.62 mev. gamma ray of Th-Pb have been investigated photographically by Ellis (22).

Henderson measured the end point of the Th C-C' and Th C'' - Pb spectra in order to determine more accurately the energy emitted around the two branches of the thorium branch point. The two branches follow the sequence:



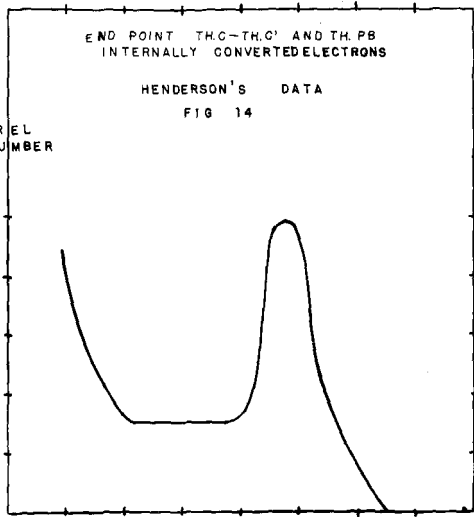
The alpha particle energies were determined by Lewis and Bowden (23) and the beta upper limits by Henderson. They are:

END POINT TH.C-TH.C' AND TH.PB
INTERNALLY CONVERTED ELECTRONS

HENDERSON'S DATA

FIG 14

REL
NUMBER



HP

ThC-C'	2.25 mev.	ThC-C''	6.20 mev
ThC' - Pb	<u>8.95</u>	ThC'' - Pb	<u>1.79</u>
	11.20 mev.		7.99 mev.

Since the energy expended by alpha and beta disintegrations around the two branches is not the same, Henderson and Ellis (24) assumed that the ThC''-Pb beta disintegration left the Th-Pb nucleus excited to the remaining 3.20 mev. This assumption proved reasonable since, among the gamma rays of Th-Pb there are two of energies 2.62 mev. and of 0.582 mev. Their energies add up to 3.202 mev. in good agreement with the expected excitation of Th Pb.

Fig. 14 shows a portion of the curve which Henderson, with a magnetic spectrograph and coincidence counters, obtained for the end point of the ThC-C' spectrum. The spectrum descends steeply, seeming to end at 2.25 mev. At 2.53 mev. there is the line of the internally converted electrons from the 2.62 mev. gamma ray. It is noteworthy, though, that the counts do not go to zero between the end of the spectrum and the line.

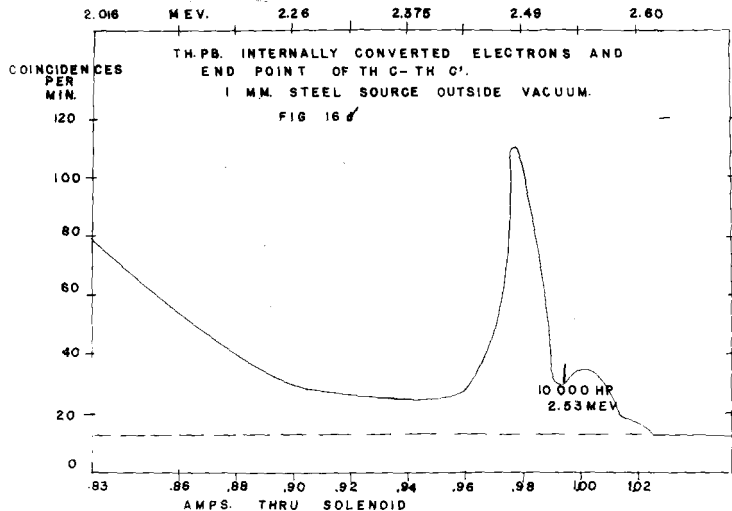
Henderson ascribed this background to scattered electrons and to recoil electrons produced in the source and in other parts of the spectrograph. Yet this explanation is not wholly satisfactory for several reasons:

- 1) If the scattered electrons are nuclear electrons from the beta ray spectra, it is strange that their distribution should

stay nearly constant and then suddenly stop. There are no scattered electrons beyond the line.

- 2) If the scattered electrons are internally converted electrons they should broaden the line. One might expect more of them to be closer to the line.
- 3) If the electrons are recoil Compton electrons from the source one would expect to find a minimum in the counts between the internally converted electrons of 2.53 mev and the maximum possible energy for the Compton electrons of 2.39 mev. Furthermore if they are Compton electrons their number should depend on the nature of the source.
- 4) If they are recoils from other parts of the spectrograph, it is surprising that their number is so great. The area under the flat part of the curve being of the same order of magnitude as the area under the line. In addition one would expect to find recoil electrons which were emitted at some point of the box above the line as well as below it.

The Institute has a source of Rd-Th suspended in a porous iron oxide gel, which has been prepared by Mr. McCoy. This source emits thoron which escapes through the gel into the air. The thoron then decays to Th A and Th B. The Th B, being ionized, can easily be collected on a negatively charged



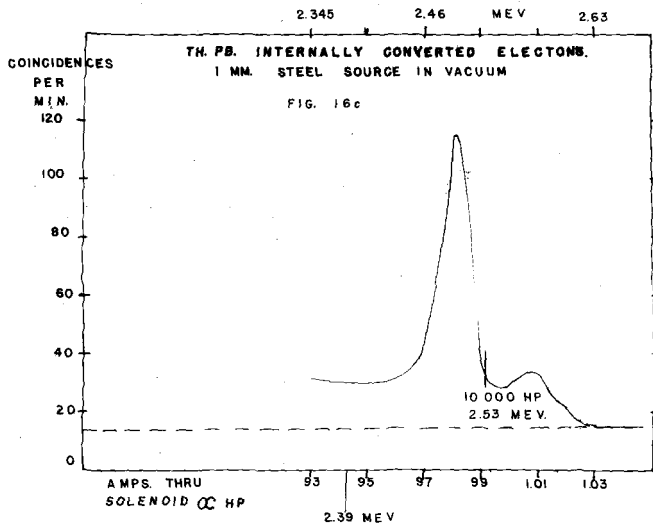


plate. In this manner a source of about 0.1 mc of 8.6 cm alpha particles can be collected. (Mr. Langmuir has measured the number of these 8.6 cm alpha particles. The branching ratio $\text{ThC}'/\text{ThC}'' = .65/.35$, so that the number of $\text{Th C}'' - \text{Th Pb}$ disintegrations corresponds to about 0.05 mc). We have used small stainless steel plates of 1 or 2 mm by 17 mm. The stainless steel is advisable because the highly ionized condition of the gas around the Rd Th gel corrodes any other metal so badly that the resultant Th B sources are no longer thin but are buried in layers of oxide. The Th B decays with a half-life of 10.6 hours. To obtain sources mounted on cellophane, a strip of cellophane was waxed on the top of the steel plate and subsequently removed.

Fig. 16 gives the curves we obtained for the end point, of $\text{Th C-C}'$ and the 2.53 mev internally converted electrons.

- a) a 2 mm. stainless steel source.
- b) a 2 mm. cellophane source
- c) a 1 mm. stainless steel source
- c) a 1 mm stainless steel source mounted in the air above the cellophane window.

All the other sources were mounted in vacuum below the window.

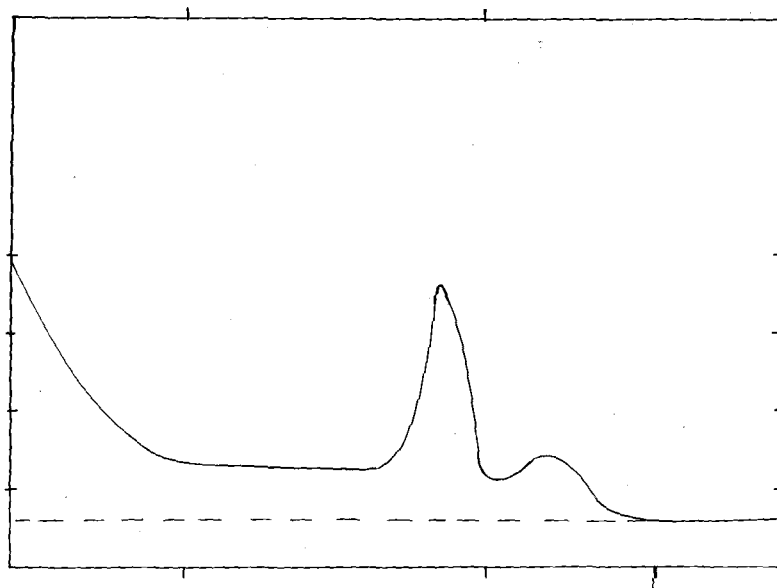
In our case, in agreement with Henderson, the counts do not drop to zero between 2.25 mev and the internal conversion line. Our spectrum does not entirely resemble this since our resolving power is greater. Our line is narrower and we have resolved the L- from the K- internally converted electrons.

Fig 16(a)

Th Pb Internally Converted Electrons and Endpoint of
Th C-C' β -spectrum

2 mm steel Source

Count/min
per Min.



2.25

2.53

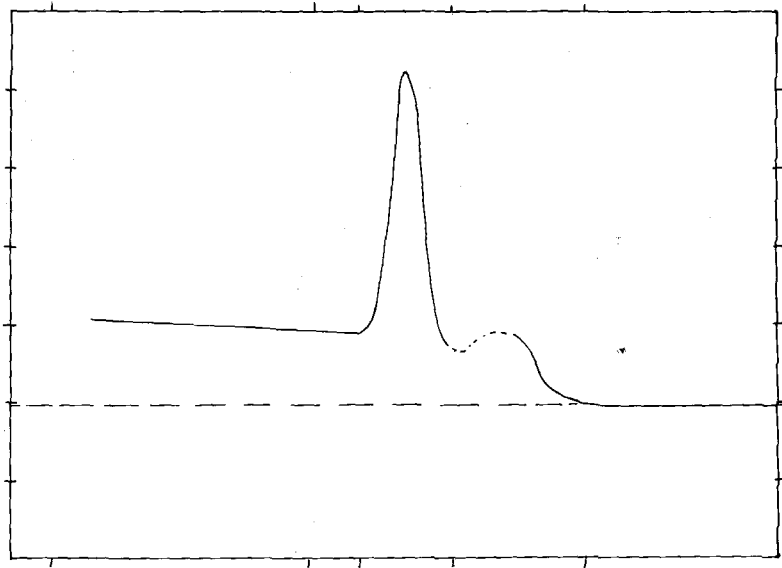
2.62

MeV

Fig 16 (b)

Th. Pb. Internally Converted Electrons.

2 mm Cellophane Source



2.25

2.39

2.53

2.62 mev.

The ratio of counts to the height of the line is the same for the stainless steel source, and the cellophane source, showing that the effect we are interested in is not due to recoil Compton electrons from the source holder. Furthermore, in no case is there a decided minimum between the tail of the line (2.45 mev) and the maximum energy of the Compton electrons (2.39 mev).

We performed one experiment not shown in the figures, to find out if the counts between 2.25 and the line increased when the pressure in the box was increased. The results are negative, showing therefore, that if the electrons are scattered, they are not appreciably scattered by the gas in the box. In Fig. 16d, the peak of the line is shifted back about 10 kev. This shift is about what we expect. 2.5 mev electrons lose about 2.1 mev/gr/cm² in water. In a mil of cellophane they would then lose 9 kev. The counts drop quite rapidly to zero above the line. The counts between 2.25 mev and the line seem to be almost constant, tapering off a little near the line.

In view of these facts we can be reasonably certain that these counts are not associated with the internal conversion line but are a tail of the nuclear beta-ray spectrum of ThC - C'. This tail may be real or it may be due to the scattering of electrons by the baffles and slits in our box.

(If most of the electrons, between the end of the spectrum and the line, are Compton recoils from the source holder, we would still have to account for the electrons between 2.39

mev and the line. It is possible that there may be an "internal Compton effect" as well as internal conversion (27) or "internal photo effect". By this "internal Compton effect" we mean a Compton effect for which momentum would not have to be conserved between the electron and the γ -ray quantum alone, but in which the nucleus of the atom emitting the quantum would get some of the momentum. The upper limit of the "internal Compton" electrons could then be the energy of the γ -ray, E_γ , rather than $E_{e(max)} = 2 E_\gamma^2 / mc^2 + 2E_\gamma$, the upper limit imposed by the energy-momentum relations for the "external" Compton effect.

Dr. Phil. Morrison (25) has calculated the magnitude of this internal Compton effect. He finds that it is only about 5 percent of the internal conversion coefficient, α_K , and would be spread over an energy range of about 100 kev.

$$\alpha_K = \frac{\text{Number of K-internal photoelectron}}{\text{Number of } \gamma\text{-quanta emitted.}}$$

The theoretical magnitude of the "internal Compton effect" is so small that we would not detect the effect.

The fact that we get the same distribution of electrons as Henderson, and the fact that the energy of the end point of the Ra E spectra and P^{32} spectra which we have measured, (see Page 68), agrees with the energy determined by Lyman (26), would indicate that if the electrons are scattered by the baffles, this scattering is independent of the particular arrangement of these baffles. This independence, with our

accuracy, is of course possible but seems improbable.

On the other hand, if the tail is real the energy balance around the two branches of the Th. branch point, would, at first sight, no longer be possible. Th. Pb., however, gives off more (28) than the two gamma rays mentioned before, which make up the level at 3.202 mev. If we were to assume one of the other gamma rays of Th. Pb to arise from a still higher excitation of Th Pb we could get levels of 3.478 mev or 3.712 mev. by adding the well-established gamma rays of 0.276 or 0.510 mev to the 3.2 mev level. If Th. Pb were left in such a highly excited state by the branch $\text{ThC-C}''\text{-Pb}$ we would have to suppose that more energy was expended in the $\text{ThC-C}'\text{-Pb}$ branch since this branch leaves Th Pb in the ground state. Thus it is not unreasonable to assume that the endpoint of the $\text{ThC-C}'$ β ray spectra is either 2.53 mev or 2.76 mev. In the second case we should be able to see the tail of the beta ray spectra beyond the internal conversion line. The lack of counts here is not conclusive evidence against the existence of such a long tail since this tail could be very weak. The 0.276 line is comparatively weak and thus if the top level of Th Pb is 3.48 mev this top level would be excited only once in 10 disintegrations. This low intensity would account for the flatness and smallness of the tail.

The supposition of this additional highly excited level in Th Pb is extremely artificial. It is, however, not essentially any more artificial than Henderson's assumption of the 3.2 mev level. It is unfortunate that this rather crucial

CALCULATED LINE SHAPES

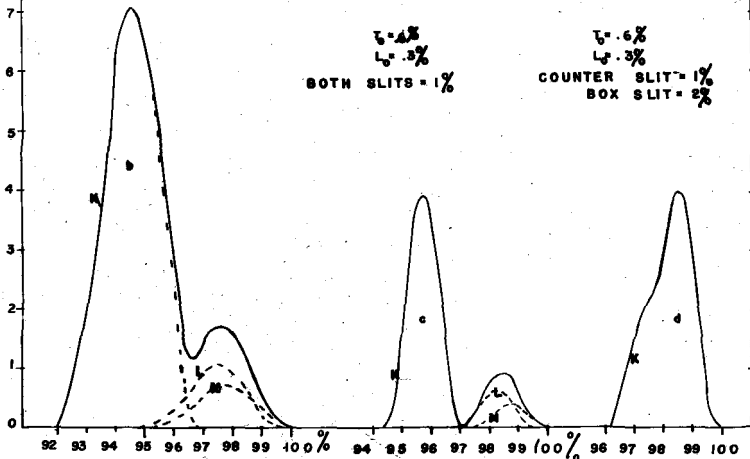
FIG 17

$$T_0 = 1\%$$

$$L_0 = .3\%$$

COUNTER SLIT = 2%

BOX SLIT = 2%



DISTRIBUTION OF
ELECTRONS AT COUNTER

$$T_0 = .6^\circ$$

$$L_0 = .3^\circ$$

$$\text{BOX SLIT} = \frac{1^\circ}{2^\circ}$$

CALCULATED LINE SHAPE

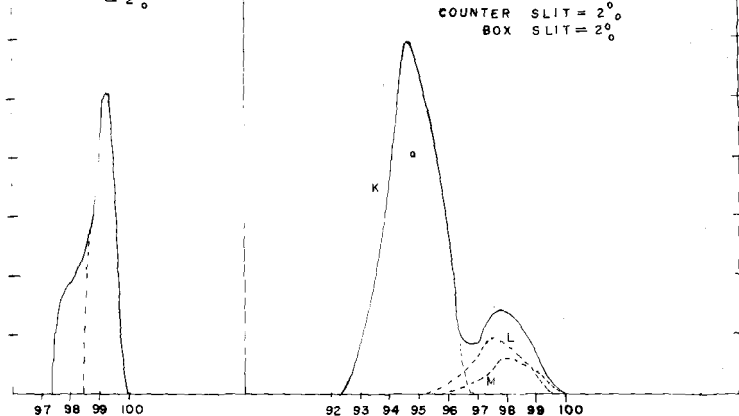
FIG 17

$$T_0 = .6^\circ$$

$$L_0 = .3^\circ$$

$$\text{COUNTER SLIT} = 2^\circ$$

$$\text{BOX SLIT} = 2^\circ$$



experiment involves at best some artificiality, if not actual ambiguity, in the interpretation of the experimental data.

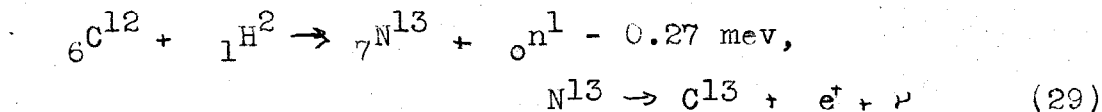
LINE SHAPE PRODUCED BY A MONOCHROMATIC GROUP OF ELECTRONS

We have calculated (appendix) the distribution to be expected due to a monochromatic gamma ray converted in the K, L, M shells of lead. This distribution of course depends on the slit width and source width, so that we have given the calculated distribution for various values of these two parameters in terms of the radius of curvature. (Fig. 17)

The agreement between calculated experimental distributions is quite good as can be seen by comparison with Fig. 16. The theoretical lines seem to be a little wider at half-maximum than the experimental lines. (This difference is perhaps due to the fact that the sensitive region in the counters may define a somewhat smaller slit than the slit in the top of the spectrograph).

NITROGEN¹³

N¹³ is produced by the bombardment of deuterons according to the reaction:

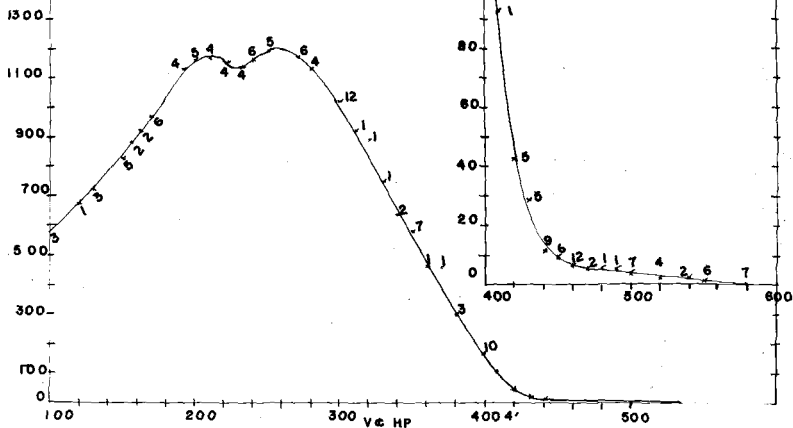


To obtain our sources Lauritsen, Lauritsen and Fowler kindly bombarded a graphite target with 10 μ -amps of 1.0 mev deuterons. The deuterons were accelerated in the pressure Van-de-Graf (30) and subjected to magnetic analysis. The graphite target was about 5 mm. thick and the bombarded surface was 3 x 17 mm². The deuterons penetrate a layer 2.5

COUNTS
PER
MIN.

NITROGEN 13
POSITRONS

FIG 18



mgs/cm² of the graphite target which has a stopping power of 25 kev for electrons. Thus, especially since most of the activity is produced nearer the surface, i.e. by the higher energy deuterons, our source is a very thin source of positrons and is suitable for determining the shape of the positron spectrum.

The half-life of N¹³ is 10.4 minutes so that most of the sources were bombarded for about 15 minutes and then quickly transferred to the magnetic spectrograph. They were mounted on top of the cellophane foil to obviate the pumping down of the box for each source. The sources produced by the Van-de-Graph varied considerably in strength but were all very strong, giving, when fresh, from 1,000 to 5,000 counts per minute at the peak of the positron distribution.

Method of Plotting

The distributions obtained are given in Fig. 18, 19, 20. For each source readings were taken at the same three values of Hp, 2500, 3750, 5000, and the intensity of all the sources was then brought to a common standard of 1,000 counts per minute at Hp = 3750. Every reading had to be corrected for the decay of the source. No readings were taken at counting rates greater than 500 per minute, in order not to necessitate a correction for counting losses. It is difficult to estimate the true statistical error, because of the enormous variation in the strength of the sources due to the initial variation and to the decay. Instead we have indicated the number of different readings taken at each point. These numbers give a rough estimate of the reliability of each point.

REL
NUMBER
PER UNIT
HP

560

480

400

320

240

160

80

0

0 1000 2000 3000 4000 5000 6000 7000

HP

NITROGEN 13

POSITRONS

FIG 19

8

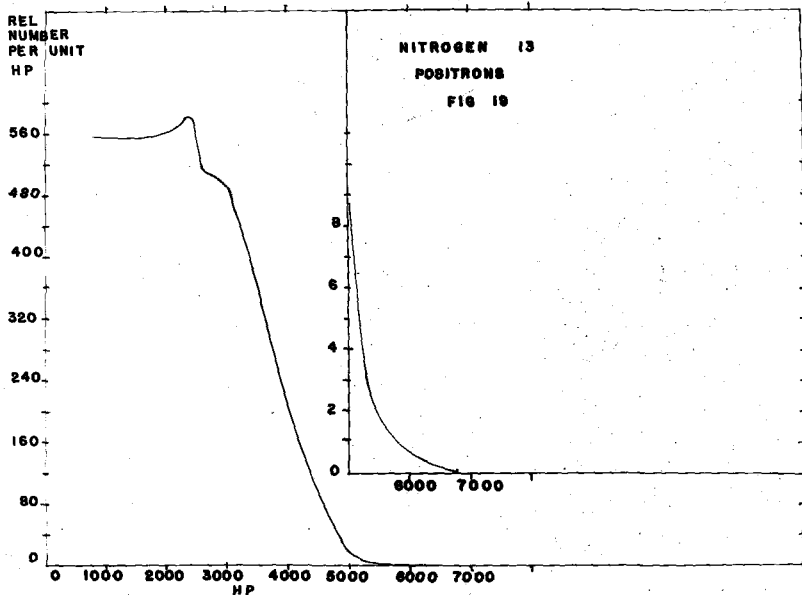
6

4

2

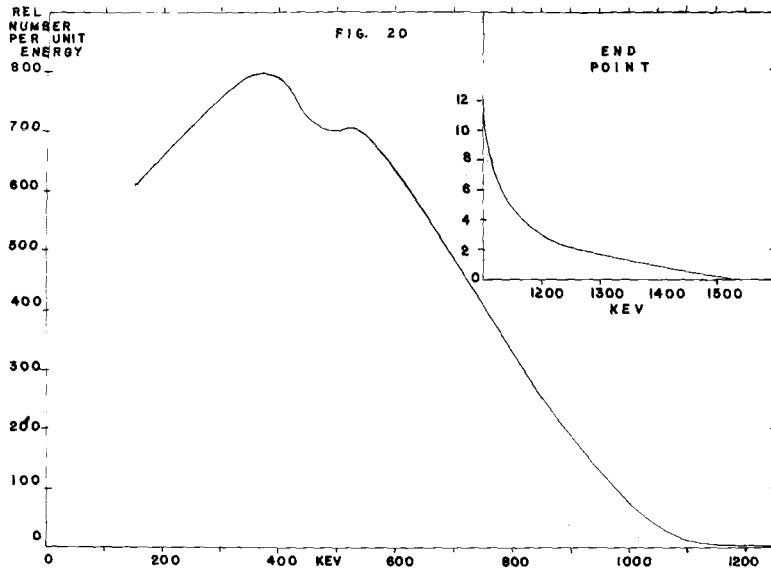
0

6000 7000



NITROGEN 13
POSITRONS

FIG. 20



Most of the readings are for 2 minute intervals.

Discussion of Positron distribution

Fig. 18 is the observed number of counts plotted against H_p . Fig. 19 is the number of counts per unit H_p . It is obtained by dividing the ordinates of Fig. 18 by H_p , and thus correcting for the change in dispersion with H_p .

Fig. 20 is the number of counts per unit energy. It is obtained from Fig. 18 by dividing by $1 + E/mc^2 = 1 + \epsilon$

$$H_p = \sqrt{(1+\epsilon)^2 - 1}$$

$$H_p d(H_p) = (1+\epsilon) d\epsilon$$

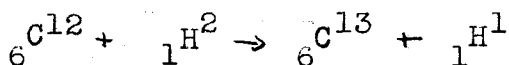
$$d\epsilon = \frac{H_p}{1+\epsilon} d(H_p)$$

Thus we divide the ordinates of Fig. 19 by $1+\epsilon$ and multiply them by H_p .

From Figs. 18, 19, and 20 it is clear that the spectrum of N^{13} is complex, the two constituents being clearly separated. The very broad maximum of the lower energy constituent, which is most clearly seen in the number-vs.- H_p curve, suggests that the spectrum may be the superposition of even more than two simple spectra.

In the case of N^{13} as in that of Th C' the spectra has a long 'tail' near the upper limit, extending from about 1.2 to 1.55 mev.

C^{13} formed by the decay of N^{13} is also formed by the direct bombardment of C^{12} with deuterons according to the reaction

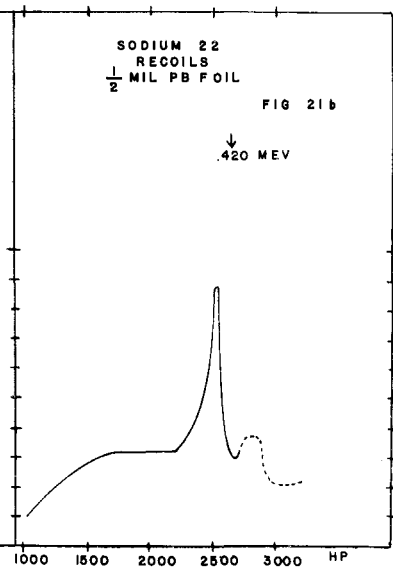
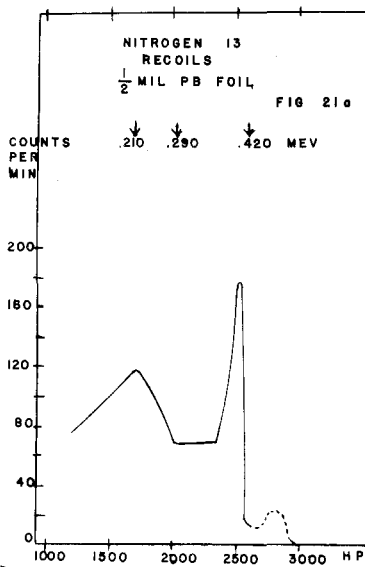


The energy of the proton group of this reaction has been measured by Cockroft and Walton (31) and the neutron energies for the corresponding reaction forming N^{13} have been measured by Bonner and Brubaker (32). The best range energy (33) relation for protons gives the value 2.30 mev for the difference in the energy liberated in forming atomic Cl^{13} by these two reactions. We would then expect the end point of the N^{13} positron spectrum to be 1.28 mev, since the value 2.30 mev was obtained by using the atomic masses. The energy of the end-point, 1.55 mev, which we obtained, is in definite disagreement with this value 1.28 mev. (We obtained the value 1.28 from the value 2.30 by subtracting 0.51 mev. for the rest energy of the positron created in the nucleus and 0.51 mev for the electron lost from the atomic shells due to the change in Z .)

The value 1.28 mev. does not correspond to the energy at which the steep part of the spectrum flattens out i.e. to the beginning of the tail, nor does it correspond to the end of it. We would thus either have to conclude that neither the beginning nor the end of the tail corresponds to the energy balance, or accept the tail as real, and find another explanation of the discrepancy in the energy balance.

Gamma Rays of N^{13} .

Richardson (34) has reported a 280 kev gamma ray associated with the decay of N^{13} . This gamma ray probably arises from an excited level in the Cl^{13} nucleus. The decay from N^{13}



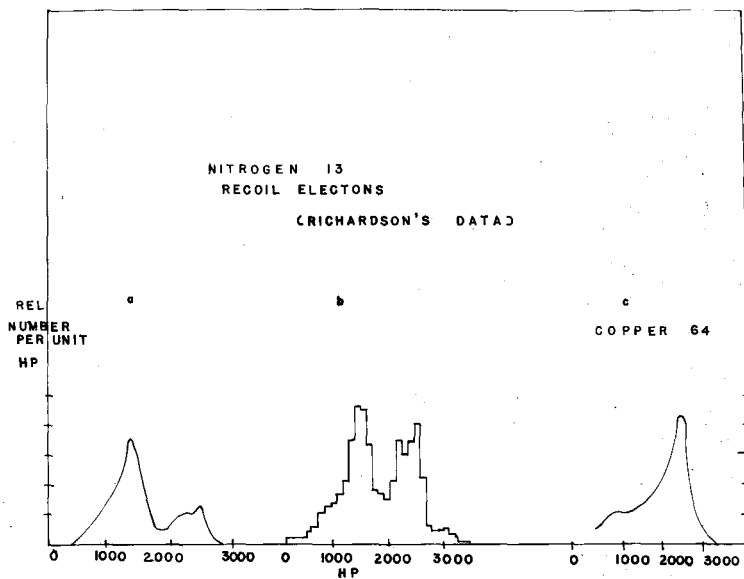
to these two C^{13} levels would then account for the complexity of the N^{13} positron spectrum.

We have looked for this gamma ray by placing the bombarded carbon target on top of a 0.5 mil Pb foil and examining the recoil electrons. The results are shown in Fig. 21 a. The recoils from the annihilation radiation appear as a sharp line, while the gamma rays from N^{13} form a broad distribution extending from 150 to 300 kev. The statistical errors involved in this distribution curve are large, but they cannot account for the difference in the shape of the distribution of the annihilation radiation and the 200 kev recoils. We have repeated the measurement of the recoils from Na^{22} (see gamma rays of Na^{22} , page 57), using a thin 0.5 mil Pb generator instead of the 12 mil one previously used. The resultant distribution is shown in Fig. 26 b. The photoeffect recoils from the annihilation radiation are identical in distribution with those obtained from the N^{13} target. The Compton recoils form the broad plateau in the recoil distribution. There is, however, no additional group of recoils as in the case of N^{13} .

Discussion of gamma-ray recoil electrons

In Fig. 21' we have given a tracing of the curve which Richardson has given for the recoils produced in a Pb foil across the cloud chamber. In 21' a the geometry was such that the positrons were not annihilated at the source. His curve does not give the true relative intensity of the gamma-ray and the annihilation radiation. From other experiments he estimates that the number of quanta per positron

Fig 21'



is 0.4. In the curves in Richardson's paper he has drawn the theoretical distribution of recoils for his foil, to show that his lines are compatible with a single gamma ray.

We have included these curves because although the 280 kev recoils, can be explained by one gamma ray their distribution is so wide that his results could also be explained on the basis of two unresolved gamma rays separated by about 50 kev.

In our case it is difficult to account for the broad group of 100-200 kev recoils on the assumption of a single gamma ray. The photo recoils from the gamma rays of this energy should give an even sharper line than is found for the annihilation radiation. The statistical errors are very large, yet the group is so broad, that it would have to be produced by at least two gamma rays if the measured electrons are all photo-electrons.

In future experiments we must determine more accurately the distribution of the recoil electrons and try to resolve various groups of electrons. If the broad group is not resolvable into several components the following discussion may provide a possible, if theoretically improbable, explanation.

(The 280 kev gamma rays may be quite largely internally converted. The internally converted electrons would get through the thin generator which was used with a loss of about 110 kev. The Pb K ionization is 90 kev, and the L

ionization is 16 kev. These three groups of electrons might make just such a diffuse distribution as we have obtained.

The photoelectric absorption coefficient for the effective layer of Pb (see page 59) is about $7 \cdot 10^{-3}$ for the annihilation radiation. In order that the internally converted electrons contribute anything to the broadening of the 280 kev line the internal conversion coefficient would have to be of the same order of magnitude as this photo-electric cross section. Using Morrison's (35) relativistic formula which neglects binding energy of the orbital electrons, i.e. $\frac{Ze^2}{\hbar v} \ll 1$ gives for

$$E = 250 \text{ kev}$$

$$Z = 6$$

$$\Delta l = 6$$

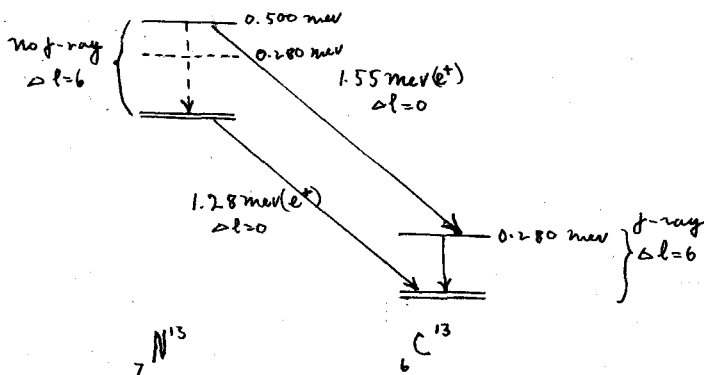
an internal conversion coefficient of 2.5×10^{-3} .

Richardson reports that there are about 0.4 quanta per positron. Thus an additional 0.35 internally converted electrons per positron would account for the approximate equality of the two components of the positron spectrum.

The energy and decay constant of N^{13} are such as to make $N^{13} - C^{13}$ an allowed transition, i.e. it falls on the first Sargent curve. Therefore, if the two levels in C^{13} differ by the 6 units of angular momentum necessary to bring the internal conversion coefficient to a large enough value, the same level in N^{13} will not decay with equal probabilities to both of these C^{13} levels. The fact that the two components of the positron spectra are of the same order of magnitude

can be seen from Fig. 20.

If there actually is internal conversion of the 280 kev gamma ray, and this will subsequently have to be verified, the two components of the positron spectrum must arise from different levels in N^{13} according to the following scheme:



The excited level in N^{13} would be either 0.280 or 0.500 mev according as the tail which we find on the positron spectrum is scattered or real.

The reaction ${}_6Cl^{12} + {}_1H^2 \rightarrow {}_7N^{13} + {}_0n^1$ is endothermic, $Q = -0.27$ mev, so that the neutron group from the 0.5 mev excited level on N^{13} would not have been observed with the 800 to 900 kev deuterons used by Bonner (36) in determining the neutron energies. On the other hand the high value of the beta ray upper limit, 1.6 mev, obtained by Kurie (36) as compared with the value 1.24 obtained by Lauritsen, Fowler and Delsasso (37), may be due to the greater bombarding energies used by Kurie. His sources were made by deuterons accelerated in the Berkeley Cyclotron.

If the excited level of N^{13} decays beta radio-actively rather than emitting a gamma ray, the level must be meta-stable. According to the calculation of Weizaker (38) an angular momentum difference of 7 would be necessary to give the excited

N^{13} state a half-life of several minutes for gamma radiation. Weizaker's values are certainly too high for low atomic numbers. The value $\Delta I = 6$ which we have found necessary to account for the internal conversion is sufficiently large to make the half-life for gamma ray decay of the N^{13} level comparable with the half-life, 10 minutes, for positron decay.

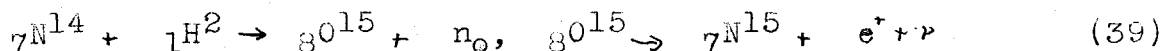
There are a number of weaknesses to this suggested explanation and level system:

- 1) The 0.5 mev internally converted electrons from the N^{13} excited state should be observed, though they should not give as strong a group as the 280 kev level of C^{13} since the internal conversion coefficient decreases with the energy of the gamma ray, and since positron decay is competing with the internal conversion.
- 2) It is remarkable that both components of the positron spectrum have the same half-life since they are between independent levels in both parent and daughter nuclei.
- 3) It is very improbable that protons be captured with comparable probability in two states differing by 6 units of angular momentum.))

Possibility of O^{15} Contamination.

It is possible that the tail beyond 1.28 mev. is due to

O^{15} contamination in the graphite target.



O^{15} has a half-life of about 2 minutes and an inspection upper limit of about 1.8 mev. The statistical errors in the counts of the "tail" are very large so that it is impossible to determine its half-life with any great accuracy. In the following table we give some of the data for the counts at energies greater than 1.3 mev. It can easily be seen from the table that a much more consistent set of values is obtained when the counts are corrected and expressed in terms of the initial intensity of the source, on the basis of a 10 minute half-life than when this correction is made on the basis of a 2 minute half life.

Source	E	count/min above background	age of source in min.	corrected for decay on the basis of	
				10' half-life	2' half-life
1	1.4 mev.	7	8	12	112
		12	14	30	1530
1	1.25	17	1	18	24
		12	5.5	17	80
2	1.37	9	11	18	430
			18	14	2000
3	1.4	8	6	12	64
4	1.3	10	1.5	11	17

The intensity of the sources was in the ratio 2/4/2/2.

When the sources were fresh no counts in the tail were greater than 20 per minute for any of the sources used.

Therefore, the counts of 100 to 2000 obtained by correcting

COINCIDENCES
PER
MIN

SODIUM 22
POSITRONS
FIG 22 b

END POINT

30

20

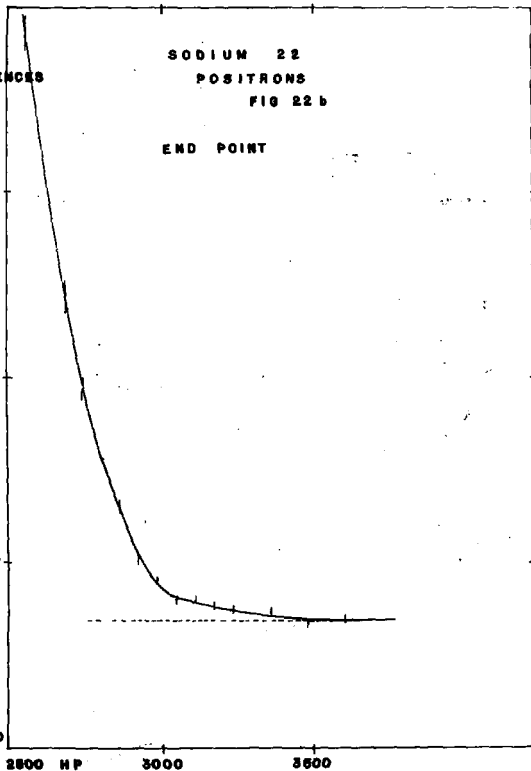
10

0

2800 HP

3000

3500



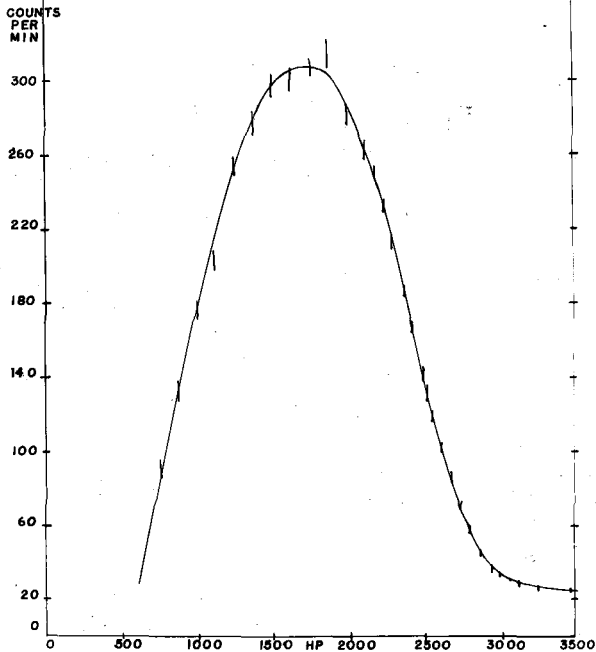
SODIUM 22
POSITRONS

FIG 22

COUNTS
PER
MIN

300
260
220
180
140
100
60
20
0

0 500 1000 1500 HP 2000 2500 3000 3500



on the basis of a 2 minute half-life are impossible.

From the data in the above table we must conclude that the "tail" is not due to O^{15} contamination. The statistical errors are so large, however, that it is not possible to completely exclude the possibility of this contamination without further experiment.

SODIUM²²

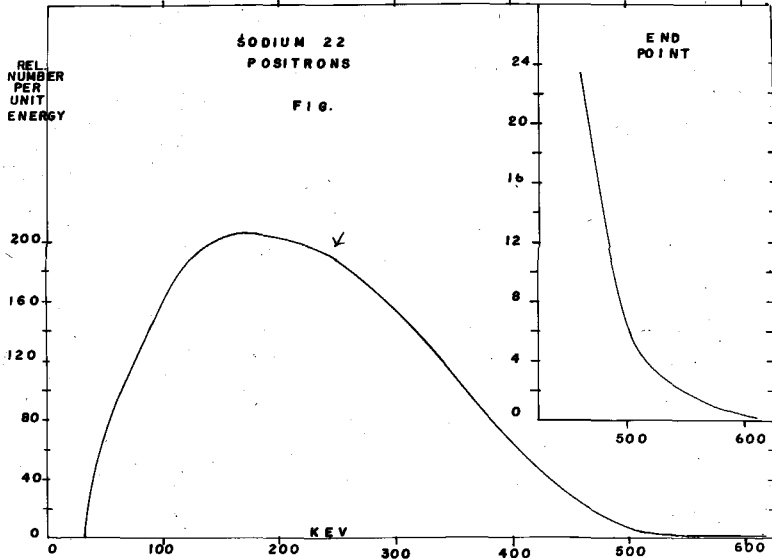
Na^{22} is a radio-active positron emitter decaying to Ne^{22} with a half-life of about 3 years. Its spectrum has not previously been examined in detail. Rough absorption measurements give an upper limit of about 0.5 mev. (40).

It can be made by the bombardment of magnesium with deuterons according to the reaction:



The sample that we have used was generously supplied to us by Louis Alvarez. It was made by 2000 μ -ampere hours bombardment of deuterons from the Berkeley cyclotron. Dr. Don Yost separated the Na^{22} from the magnesium and prepared it in the form of NaCl. We have mounted a thin layer of this NaCl on a strip of cellophane 2 x 17 mm. to use as a source for examining the energy distribution of the emitted positrons.

Fig. 22 and 23 show the distribution of positrons. Fig. 22 gives the observed distribution plotted against Hp and Fig. 23 gives the distribution per unit energy.



Discussion of Positron Distribution.

The distribution shown in Fig. 18 and 19 both suggest that the positron spectrum of Na^{22} is not simple but is made up of two spectra, the peak of the two distributions differing by about 50 kev. The higher energy spectrum would have to be considerably weaker than the lower energy one to account for the shape of the distribution. The two components of the distribution are not clearly resolved as in the case of N^{13} . The distribution that we have obtained might equally well be the shape of a "simple" spectrum. There is however a hump in the distribution (indicated by the arrow) which suggests a complex spectrum. If this hump were instrumental or statistical, that is, if in reality there were no such hump, then the true distribution would have to miss about six adjacent intervals near the peak of the distribution.*

* The errors given in Fig. 18 are what we have called the 0.8 errors and are about twice the probable errors.

The 0.8 errors give an interval such that the chance of the true value lying within the interval is 0.8. For the probable errors, the chance that the true value lies within the interval is 0.5.

L. I. Schiff (41) provided the following convenient table. Let $P(\Delta N, N)$ be the probability that the true value of a quantity determined by N counts lies between $N + \Delta N$ and $N - \Delta N$. If x is defined by

$\Delta N = x \sqrt{2N}$ then x is given for different values of p by

P	.9	.8	.7	.5
x	1.163	.906	.733	.477

If the N counts are obtained by counting for T minutes the error in the number of counts per minute will of course be $\Delta N/T$.

The values of x given above are the limits of integration on Gauss's error integral for which the area under the error function is equal to P .

In using the .8 errors, therefore, it is quite unlikely that the true distribution fails to pass through more than two or three of the intervals out of every ten points.

$\sqrt{\frac{N}{1}}$

SODIUM 22
POSITRONS
FIG 24

140

100

60

20

0

1.0

1.5

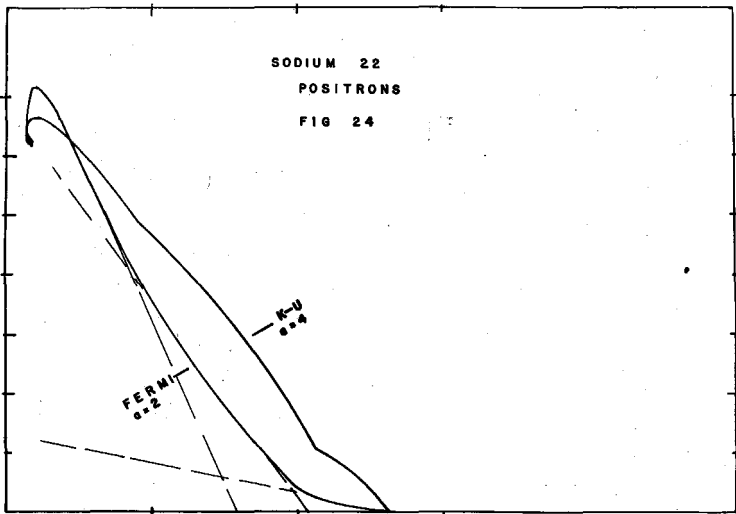
2.0

2.5

FERMI
 $\alpha=2$

K-U
 $\alpha=4$

$\sqrt{1+\eta^2}$



The omission of a smaller number of points might, of course, alter the shape of the distribution but it would not change the necessity of assuming either an unusual shape for a simple spectrum or that our distribution consists of two spectra.

The Fermi (42) plot of the distribution, shown in Fig. 24, supports the conclusion that there are two spectra whose upper limits differ by about 100 kev. This Fermi plot indicates that there is still a third and weaker spectrum whose upper limit is 0.6 mev. The interpretation of this "tail" is open to the same ambiguities as those that we have discussed in connection with the Th C-C' and the N^{13} spectrum. The Fermi theory may be far from correct so that we cannot place too much weight on its implications. Even if the tail is real and not instrumental, we would not be forced to consider it as due to an independent spectrum. Our whole distribution may be that of a simple spectrum despite the distinct components indicated by the Fermi plot.

The Gamma Rays of Na^{22}

To investigate the gamma rays emitted by Na^{22} we used a thick target (all of the available sample) on top of a 12 mil lead strip, 2 x 20 mm. The NaCl was held in place on the lead by wetting it with some very dilute shellac. This lead recoil electron generator, has a stopping power of about 700 kev. so that electrons which are generated in the top layers of it will be slowed down by the lead and thus be measured as electrons of from 300 to 700 kev less energy, depending on their initial energy. Thus the lead generator acts as a "thick target". For the purpose of finding gamma rays,

SODIUM 22
RECOILS
12 MIL PB FOIL

FIG 25

COUNTS
PER
MIN

200

150

100

50

0

420

509

1.005

1.190

1.285

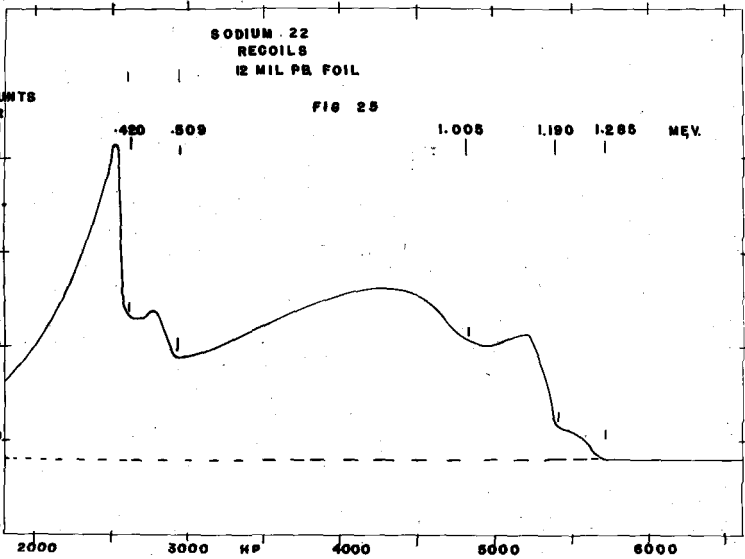
MEV.

HP

4000

5000

6000



whose energy is to begin with completely unknown, such a thick target is much more satisfactory, since the existence of a gamma ray is then evident over a broad range of field settings.

We have examined the spectra of the recoil electrons up to 6 mev and have found nothing above 1.3 mev. The distribution is given in Fig. 25. From this figure it is clear that there are two gamma rays at about 1.3 and 0.5 mev. The second of these is the annihilation radiation from the positrons.

Discussion of Recoil Electron Distribution.

The apparent complexity of the recoil electron spectra is due to the fact that for one gamma ray we get two groups of photoelectrons and a wide band of Compton electrons. The two groups of photoelectrons are due to the photo-effect in the K-shell of Pb and in the L-, M-, N-, shells of Pb. The ionization potentials are $K = 90$ kev, $Li = 13$ kev, $Mi = 3$ kev, $Ni = 2$ kev. The photo-effects from the L, M, N, shells are not resolved.

At first sight one would expect the photo-effect distribution to give a sharp rise in the number of counts 90 kev (the K- ionization potential) below the energy of the gamma ray and that the number of counts would then remain constant for lower energies. This picture is only approximately true for the 1.3 mev gamma ray and is not at all true for the 0.5 mev annihilation radiation. The explanation of the dropping off of the number of counts at lower energies is as follows:

The H_p interval which is collected at any value of H_p is about 2 percent of that H_p . Therefore, even if the distribution of electrons against H_p were constant there would be a linear decrease in the number recorded. This linear decrease in the number of counts can be seen in the case of the 1.3 mev gamma ray. In the case of the 0.5 mev gamma ray, another effect also contributes to the decrease in the number of counts. If at any value of H_p we collect a band of electrons which have lost 2 percent of this H_p these electrons must have been ejected by the gamma ray from a layer of lead whose thickness is just that in which electrons lose 2 percent of their momentum, H_p being proportional to momentum. This thickness is of course a function of the momentum of the electron. For high energy electrons the range varies almost linearly and not very rapidly with momentum, while for low energy electrons this variation is much more rapid and is no longer linear (43). Thus if we considered a surface layer of Pb, of thickness t , such that electrons will have lost 10 kev. in this layer, and then consider a layer in the Pb, at a depth, x , of thickness t , the electrons from the front of the layer will have travelled a distance x while those from the back will have travelled a distance $x + t$, These two groups will now differ in energy by much more than 10 kev. By the time they have reached the surface layer t , they will have both been slowed down say to 0.3 mev, so that in the same layer, t , they will now lose much more energy. Thus at 300 kev, not only do we collect a smaller spread of

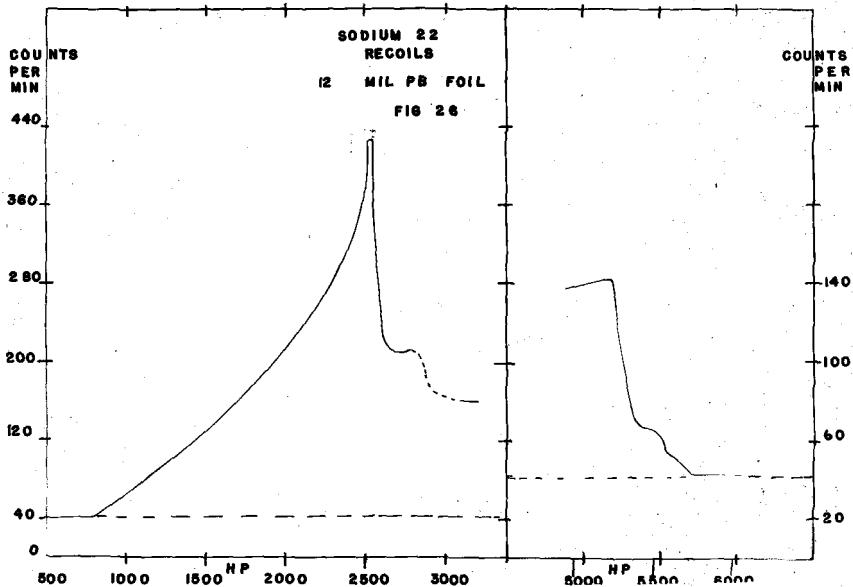
momentum, but electrons with the same spread in momentum have been generated in a much thinner layer of Pb than those at 500 kev.

Relative Intensity of 0.5 mev and 1.3 mev Gamma Rays.

The distribution shown in Fig. 21 was obtained with a 1 mil copper foil separating the counters from the vacuum in the spectrograph box. This foil has a stopping power for electrons of about 150 kev so that some of the electrons at 420 kev were absorbed in the foil. These losses would have to be corrected in determining the relative intensities of the 0.5 and the 1.3 mev gamma rays.

It is extremely difficult to correct for these losses since they arise from two different processes. Some of the electrons are actually stopped or reflected in the foil while others are merely scattered. The data on absorption therefore obtained by other men such as Madgwick (44), or Varder (45), or Schonland (46), is not applicable unless they not only use the same absorbing material, but also collected the electrons with the same solid angle as we do. We therefore, tried to calibrate our foil by using the known relative intensity of some of the low energy Th C and C" gamma rays. The internally converted electrons from these gamma rays have been measured photographically by Ellis (47). They are, however, superimposed on a very intense continuous beta ray spectra and it is therefore difficult to obtain reliable intensity measurements with counters.

We, therefore, abandoned the effort to calibrate our copper foil and instead replaced it with a cellophane foil



with a stopping power of 40 kev and repeated the measurements on the intensity of the photo-effect for the two lines.

Our results are given in Fig. 26. The thick target completely obliterates the Compton effect from the 0.5 mev annihilation radiation.

To calculate the relative intensity of the 0.5 and the 1.3 mev gamma rays we must take into account the following facts:

- 1) The photo-effect is different for the two energies.
- 2) The spread in Hp is different for the two energies.
- 3) The layer of Pb in which electrons lose the same amount of energy is different.
- 4) Two quanta are emitted for every positron annihilated.

The calculations are given below.

Actual intensity of electrons.

<u>mc²</u>	<u>1.3 mev</u>
426 counts/min. Peak of K Line	142 counts/min.
<u>-208</u> counts/min. Head of K Line	<u>- 65.5</u>
218 ± 6	76.5 ± 2.5

Width of energy band collected by slit
at one field setting

Calculation of relative intensity
of 0.51 and 1.3 mev gamma rays (cont.)

Energy loss per cm. in lead

for electrons of

mc²

1.3 mev.

1.45 mev/gr/cm²

1.1 mev/gr/cm²

Layer in which electrons lose

10 kev

26 kev

6.9 x 10⁻³ mgs/cm²

24 x 10⁻³ mgs/cm²

Photo-electric coefficient

by Read's empirical formula.

1.018

0.188

Number of 1.3 mev quanta per 0.51 mev quanta

$$76.5 \times 6.9 \times 1.018 / 218 \times 24 \times 0.188 = 0.55$$

Number of 1.3 mev quanta per positron.

$$2 \times 0.55 = 1.1$$

In this calculation the difference in the angular anisotropy of the photo-effect at 1.3 and 0.51 mev has not been taken into account. This correction has been omitted because it will be small. The Pb generator is in contact with the Na²² and thus subtends a large angle.

The ratio of the number of 1.3 mev quanta to the number of positrons is then 1.1 ± 0.1.

The electroscopic activity of our Na²² source was the same as that of about 0.1 mc. of radium. Using Gurney's (48)

value of 5 gamma rays for every alpha particle of Ra, our source gives about 6.10^8 , 1.3 mev gamma rays per minute. Using Read's value of 0.19 as the photo-electric cross section per cm., and taking into account,

- 1) Only one-half of the gamma rays go through the lead generator.
- 2) A solid angle of 5.10^{-3}
- 3) The electrons are generated in a layer in which they lose 26 kev.
- 4) The gamma rays, since the Na^{22} is in contact with the lead, traverse this layer obliquely as well as perpendicularly.

We would expect to get about 100 electrons per minute from the 1.3 mev line. Actually we get 76. The agreement is not very good but is certainly satisfactory.

The last effect (4) was taken into account by multiplying the thickness of the layer of Pb in which 1.3 mev electrons lose 2 percent of their energy by 4.5 which is approximately the average of cosecant over the angle subtended at the source by the Pb generator. The value 4.5 neglects the angular anisotropy of the photo-effect.

Summary of Na^{22}

If the separation of the two components of the positron spectra is 100 kev as indicated by the Fermi intercepts, then we should have detected this 100 kev gamma ray by the recoil electrons from the lead generator. If, however, the separation of the two components is only about 50 kev, as

suggested by the separation of the peak and the hump of the distribution, then these recoil electrons would have been almost wholly absorbed in the cellophane foil below the counters. We hope to repeat this experiment in the near future with a still thinner foil between the counter and the box, and also to look for 50 or 100 kev internally converted electrons from the source. If we fail to find either the gamma ray or the internally converted electrons it will be difficult to believe the suggested complexity of the positron spectrum.

The comparison of the relative intensities of annihilation radiation and 1.3 mev gamma rays is of interest because of its bearing on the process of K-capture.

There are two possibilities for the decay of a radioactive positron emitter. It may either emit a positron and a neutrino which share the energy in a continuous manner, or it may capture one of the orbital K electrons emitting a neutrino of fixed energy. In the first case 0.51 mev of the decay energy is used in creating the positron, while in the second case an additional 0.51 mev is added to the decay energy which is carried away by the neutrino. This 1 mev extra energy makes the ^{probability of the} K-capture process comparable ^{that of} with the positron decay process for low energy spectra, and in some cases the K-capture may occur when positron decay is energetically impossible.

Willys Lamb (49) has calculated the ratio, M , of the

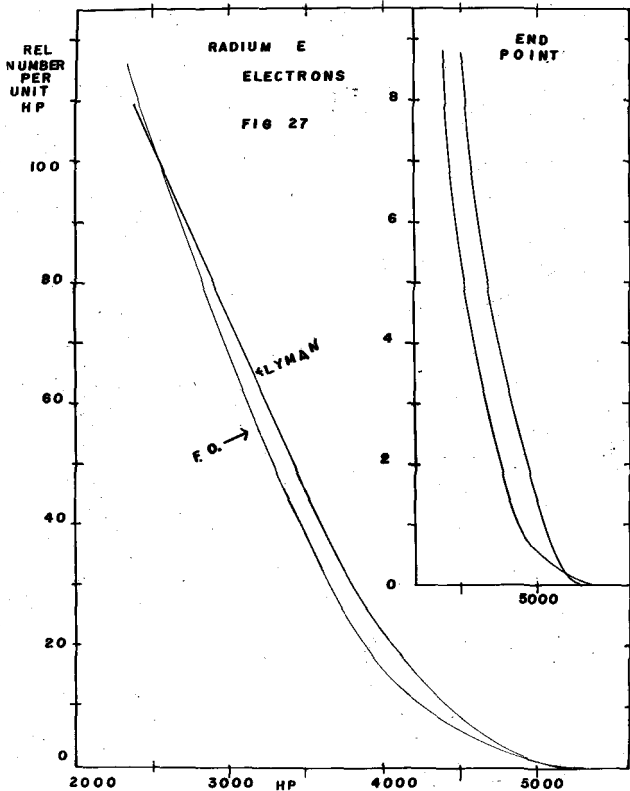
number of K electrons captured to the number of positrons emitted. The ratio M depends on the particular form of the coupling of the electron-neutrino field with the nuclear field, and in addition, on the change in the total angular momentum of the nucleus during the radioactive transition. The following table gives the values of M obtained by Lamb using the Fermi and the K-U (56) ansatze, for $\Delta\ell = 0$, and 1. These values were calculated for sodium 11 and the upper limit energy of 0.5 mev. The experimental value for the endpoint of the positron distribution is 0.55 mev.

Table 4		
$\Delta\ell$	F	K.U.
0	1.2	4.7
1	2.1	30.4

Our value of 1.1 is in good agreement with the value he has obtained on the basis of the Fermi theory with $\Delta\ell = 0$.

We cannot be certain, however, that this value of 1.1 is really the K capture ratio, for this involves the assumption that the 1.3 mev gamma ray is given off only in the case of K capture. That is, it involves the assumption that when a positron is given off the resultant neon nucleus is left in the ground state, while when a K electron is captured, the neon nucleus is left in an excited state of 1.3 mev. The remaining 0.5 mev would then be carried away by the neutrino.

Although this assumption is reasonable it is not inevitable.



The gamma ray may either be associated with both K capture and positron emission or only with positron emission. It would perhaps be worth while to record the number of coincidences between a counter placed at the source and one collecting the positrons. The number of random coincidences would be high but it would be possible, with the proper geometry, to determine whether the emission of a gamma ray were coincident with the emission of a positron. The annihilation radiation would contribute to the random coincidences but not to the true ones since a positron would not be both recorded and annihilated in the target.

We did not perform this experiment, however, because it would have involved a considerable change in our set-up and because Dr. Alvarez plans to perform a more reliable one. He will measure spectroscopically the amount of neon developed by the decay of the Na^{22} and compare this to the number of positrons emitted by the source. These measurements will, of course, give a value for the K capture ratio which although probably not very accurate, will at least distinguish between the three possibilities mentioned above, thus making definite the application of our more accurate value for the relative gamma ray intensities.

Radium E

In Fig. 27 we have given the distribution of electrons near the end-point of Ra E. The measured distribution has been corrected by dividing the ordinates by H_p . Our end-

REL
NUMBER
PER
UNIT
HP

PHOSPHORUS 32

ELECTRONS

FIG 28

200

150

100

50

0

LYMAN →

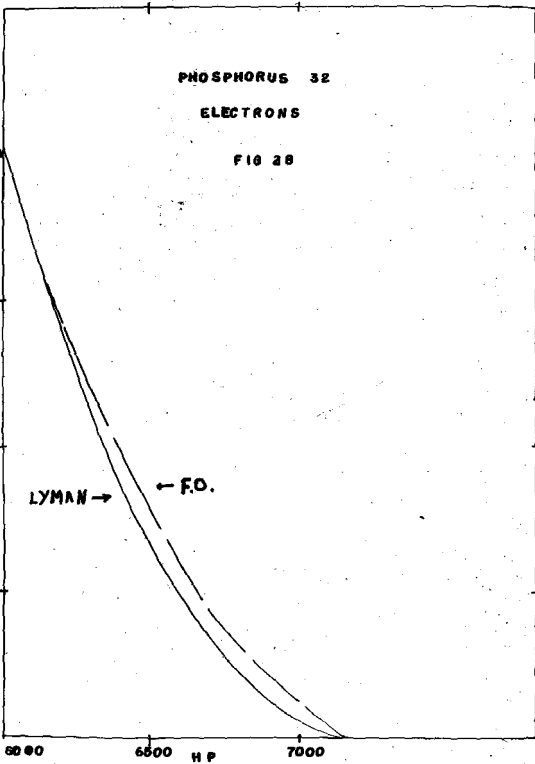
← FO.

6000

6500

HP

7000



point of 5350 ± 50 gauss cm. agrees with Lyman's (50) value of 5280 ± 20 gauss cm. The shape of our distribution near the end-point is, however, not the same as Dr. Lyman's. Ours flattens out considerably more than his, showing a more pronounced "tail". The difference, however, is not very great and is perhaps due to our greater resolving power.

Phosphorous³²

In Fig. 28 we have given our and Lyman's (50) distribution for the end-point of the P³² electron spectra. Our value for the end-point of 7150 gauss cm. agrees exactly with his. In this case our distribution is steeper than that of Lyman. But this steepness is misleading since we were unable to obtain a really thin target of P³². Our sample was made by bombardment with 2 mev deuterons on the Berkeley cyclotron and was in the form of sodium acid phosphate mixed with a large amount of non-radioactive phosphate. In order to obtain sources of sufficient intensity they had to be 0.05 to 0.1 mm. thick, thus slowing down the 1.7 mev electrons about 50 kev, and making the distribution at the end-point appear steeper than it really is.

The Fermi and K-U plots of the beta-ray spectra according to the formula

$$\left(\frac{N}{\pi_i}\right)^{\frac{1}{\alpha}} = K \left\{ \sqrt{1 + \pi_{e_{max}}^2} - \sqrt{1 + \pi_e^2} \right\}, \quad \pi_e = \frac{HP}{1700}$$

$\alpha = 2$ Fermi
 $= 4$ K-U

are not very sensitive to the shape of the spectra. We have plotted our P³² results according to these two methods. The results are identical with those of Lyman, the Fermi plot giving a straight line near the end-point while the

K-U plot is markedly concave towards the origin.

Discussion of Phosphorous³² and Radium E.

The agreement of our value for the end-point of P^{32} and Ra E with that of Lyman, as we have already mentioned, gives considerable credibility to the tails that we have found for most of our distributions. At least it is hard to understand this agreement if the "tails" are instrumental, since our spectrograph differs considerably from that of Lyman.

On the other hand, in the two cases where an independent value for the energy of the end-point exists, i.e. in the case of Th C-C' and N^{13} we have been forced to adopt very artificial and unsatisfactory assumptions about the levels of the N^{13} and Th-Pb nuclei to account for the "tails", that is for the high value for the end point of the continuous beta ray spectra.

The questions and ambiguities opened by this preliminary work can only be solved by further experiments.

Yet it is evident, even from the incomplete set of experiments that we have described, that many and probably most beta ray spectra are complex. It is not surprising, therefore, that the distributions hitherto obtained do not fit the distribution predicted by the simple theory of Fermi. Furthermore, it is evident, since the ground state of a radioactive nucleus can decay to various states of the daughter nucleus, that the decay constant depends not only on the upper limit and on Z but must vary in a somewhat irregular manner. The allowed radioactive transformation with the greatest

energy will, it is true, be the major factor in determining the decay constant, but the other possible transformation will also contribute to the decay of the parent nucleus.

In concluding we will simply enumerate some of the further experiments which this preliminary work has suggested.

VALUE OF THE END POINT

- 1) We must obtain/^avery strong source of P^{32} or Ra E. It is possible that with a stronger source we may find a tail extending beyond the present value of our end point.
- 2) We must examine the region at the head of the Th.Pb. internal conversion line. We may find evidence of a tail, produced by the line itself or being a continuation of the "tail" which occurs behind the line.
- 3) We must determine more accurately the half-life of the N^{13} end point distribution, or see if the tail disappears when we use a diamond, and thus a N^{14} free, target.
- 4) We must try to find whether or not the N^{13} tail, and thus the excited N^{13} state, disappears when the bombarding energy is lowered. This experiment is difficult to perform since the excitation function for the production of N^{13} from C^{12} decreases so rapidly with the energy of bombardment.
- 5) We must compare the positron distribution

and the gamma rays of C^{11} (52) and O^{15} with those of N^{13} . If our value for the C^{11} end point agrees with the value obtained from the Q of the reaction, considerable credence would have to be given to our value for the N^{13} end point.

- 6) We must alter the position of the baffles and the material around the source to see if any change in the relative intensity of the "tails" and the rest of the distribution is produced.

GAMMA RAYS AND COMPLEX SPECTRA

- 1) We must look for a low energy gamma ray or internally converted electrons from Na^{22} to determine whether the positron spectrum is complex or not.
- 2) We must try to resolve the broad group of N^{13} recoil electrons, determine their half-life, and in addition look for internally converted electrons produced in the source.

Although the experiments outlined above will probably remove most of the ambiguities that we have considered, the more general questions about beta ray spectra, which motivated our work at the outset, will only be partially answered.

Among our future experiments we would like to include an investigation of the high energy electron spectra of Li^8 (57) and B^{12} (58) and also of some of the allowed and

non-allowed spectra of the heavier elements. We hope that these experiments will make clear the role played by the many factors that determine the shapes of beta ray spectra, and thus that they will ultimately throw more light on the nuclear processes in beta-radio-active transformations.

APPENDIX

Calculation of the intensity distribution produced
by a finite source of monochromatic electrons.

1) POINT SOURCE

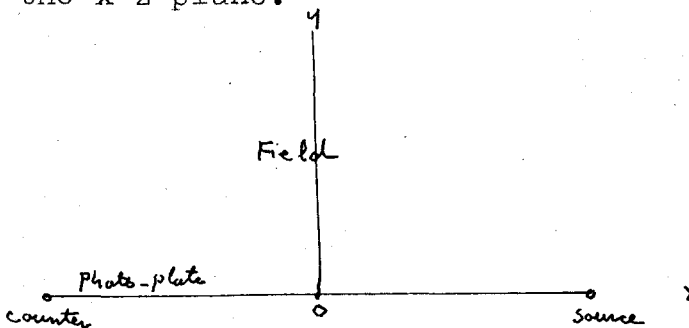
In a uniform field, H , electrons travel in a circle of
radius, ρ , such that:

$$H\rho = \frac{mc}{e} (1 - \beta^2)^{-\frac{1}{2}} = M$$

where M is the component of the momentum perpendicular
to H .

Let us consider only those electrons emitted by the
source in a plane perpendicular to H .

Let $y = 0$ be the lower limit of the magnetic field in
which the source and detector are placed. Let the
source be at $x = \rho$, $y = 0$. Then we will calculate the
number of electrons per unit length falling on the negative
 x -axis. e.g. the blackening along the x -axis of a photographic
plate in the x - z plane.



The equation for all circles going through the source is

$$(x-a)^2 + (y-b)^2 = \rho^2, \text{ with } x = \rho \text{ when } y = 0$$

or in polar coordinates with the same origin,

$$(r \cos \psi - a)^2 + (r \sin \psi - b)^2 = \rho^2$$

when $\psi = 0$ then $r = \rho$

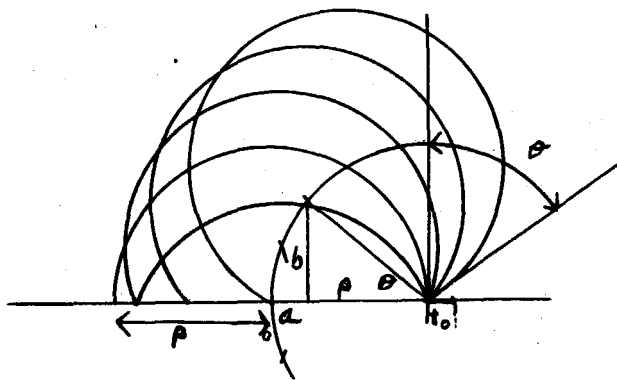
so that $r^2 - 2ar + (a^2 + b^2) = \rho^2$ or $b^2 = 2ar - a^2$.

Using this value of b , we get, when $\varphi = \pi$

$$b^2 + r^2 + 2ar + a^2 = \rho^2$$

$$r = \rho - 2a \quad \text{since } r > 0$$

If the electrons leave the source at all angles, with the normal to the plane $y = 0$, from $\theta = -\frac{\pi}{2}$ to $\theta = +\frac{\pi}{2}$, the center of all the paths of the electrons leaving the source will lie on a circle of radius ρ whose center is at the source.



Then $a = \rho (1 - \cos \theta)$. Thus when $\varphi = \pi$, $r = (\rho - 2a) = \rho (2 \cos \theta - 1)$

Therefore, for any value θ the electrons emitted at $+\theta$, and $-\theta$ will fall on the same point of the photo-plate. For a point source, no electrons starting at different values of $|\theta|$ will fall on the same point of the photo-plate.

If the angle at which the electrons leave the source is limited to some value θ_0 by a slit near the source, then the band on the photo-plate will be limited to $2\rho(1 - \cos \theta_0)$

in width. In our case $\theta_0 = 8^\circ$ and the band is 2 percent of ρ .

We must now calculate the density per unit r which is produced by a distribution of electrons isotropic in θ at the source.

Let the number per unit θ be n

The number in $d\theta$ will be $\frac{nd\theta}{dr} = \frac{n}{2\rho \sin\theta}$

since $r = \rho(2 \cos\theta - 1)$

Thus the density $D(r)$, or the blackening per unit r is

$$(1) \quad D(r) = \frac{n}{[4\rho^2 - (r + \rho)^2]^{\frac{1}{2}}}$$

Let us now consider a source of zero length perpendicular to the paper and of width t_0 along the x axis.

For any point, t' , of our source there is an angle, θ' , such that $r(\theta_0, 0) = r(\theta', t')$

Thus for these values of θ_0 , θ' and t

$$2\rho \cos\theta_0 - \rho = 2\rho \cos\theta' - \rho - t$$

or $t = -2\rho(\cos\theta_0 - \cos\theta_1)$ and $\cos\theta' = \cos\theta_0 + \frac{t}{2\rho}$

As before, the density due to the electrons leaving the source at (t', θ') will be:

$$d(rt\theta) = \frac{ndt}{2\rho \sin\theta'} = \frac{ndt}{2\rho \{1 - (\cos\theta_0 + t/2)\}^{\frac{1}{2}}}$$

$$\frac{ndt}{2\{4\rho^2 - (\rho + \rho + t')\}^{\frac{1}{2}}}$$

However, the density at some value of r will in this case also get contributions from electrons which leave the source at $t < t'$ and the corresponding $\theta > \theta'$. For any value of r there will be contributions to the density

for all values of $t \leq t_r = \rho - r$. For t_r , $\theta' = 0$.

Since t_0 is the largest value t can take

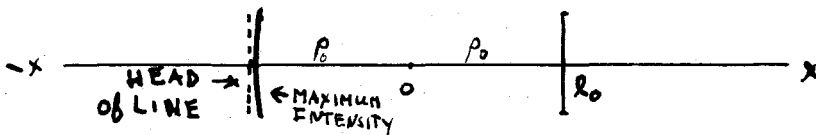
$$D(rt_0) = \int_0^{t=\rho-r} \frac{ndt}{\sqrt{4\rho^2 - (r + \rho + t)^2}} = \cos^{-1} \left(\frac{r + \rho}{2\rho} \right) \text{ for } \rho - r < t_0$$

(2)

$$= \int_0^{t_0} \frac{ndt}{\sqrt{4\rho^2 - (r + \rho + t)^2}} = \cos^{-1} \left(\frac{r + \rho}{2\rho} \right) - \cos^{-1} \left(\frac{r + \rho + t_0}{2\rho} \right) \text{ for } \rho - r \geq t_0$$

We must now consider the distribution produced by a source of finite length, l_0 , perpendicular to the plane of the paper as well as of finite breadth, t_0 .

Let us consider only the density along the center of the photo-plate, that is along the negative x-axis. Electrons with a given (θ, t, M) which start from some point of the source not on the x-axis will fall on the photoplate nearer the origin than those starting with the same (θ, t, M) but from a point of the source on the x axis.



The electrons still have the same momentum, M , but the projection of M on a plane perpendicular to H is smaller. Thus the projection of their paths on this plane will be a

circle whose radius $\rho < \rho_0$. Although the head of the line will be at $-x = \rho_0$, the maximum number of electrons will fall at a smaller value of x than in the case where $l_0 = 0$. Although we will calculate only the distribution along the $-x$ axis the source will in reality produce a line on the plate. The head of this line will be straight and parallel to the z axis at $x = -\rho_0$. But the line will appear curved. At the center of the line the minimum value for the radius of the projected circle, ρ , will be

$$\sqrt{\rho_0^2 - l_0^2}.$$

While at the edges of the line, the minimum value of will be $\sqrt{\rho_0^2 - 4l_0^2}$. Thus the maximum density will be further from the head of the line, $x = -\rho_0$, at the edges than at the center.

We must, to get the distribution along the x axis, add up the distributions of electrons which leave the source at different points (l, θ, t) . We can replace l with the angle ψ since there is only one value of l for any value of ψ for which the electrons will fall on the x axis at the photoplate. Since the electrons are emitted isotropically in ψ the number reaching the $-x$ axis per unit l will not be constant but will vary as $\cos \psi$.

In our case $l_0 = 5$ mm., $t_0 = 2$ mm., $2\rho_0 = 400$ mm. We have neglected the variation in $\cos \psi$ as l goes from 0 to l_0 , i.e. we have taken the number of electrons emitted per unit l as constant.

As a further slight approximation we have added together the distributions due to a source of width t_0 with radii ρ ,

varying from ρ_0 to $\sqrt{\rho_0^2 - l_0^2}$. More correctly we should have added together the distributions produced by source of width $t_0/\cos \psi$. Here also, we have neglected the variation in $\cos \psi$ and have set $\cos \psi = 1$.

Setting $r = x\rho$

$$t = m\rho$$

$$\sqrt{\rho_0^2 - l_0^2} = s\rho$$

we get for the density $D(r, t_0, l_0)$ the following integrals with the appropriate limits of integration.

$$\begin{aligned} D(r, l_0, t_0) &= \int_{\rho_0}^r \cos^{-1}\left(\frac{r+\rho}{2\rho}\right) d\rho \quad \text{for } |x| \geq \sqrt{1-s^2} \\ &\quad \text{and } |x| > 1-m \\ &= \int_{\rho_0}^{s\rho_0} \cos^{-1}\left(\frac{r+\rho}{2\rho}\right) d\rho \quad \text{for } |x| \leq \sqrt{1-s^2} \\ &\quad |x| > 1-m \end{aligned} \quad (3)$$

$$\begin{aligned} &= \int_{\rho_0}^{s\rho_0} \cos^{-1}\left(\frac{r+\rho}{2\rho}\right) d\rho - \int_{\rho_0}^{s\rho_0} \cos^{-1}\left(\frac{r+\rho+t_0}{2\rho}\right) \\ &\quad \text{for } |x| \leq \sqrt{1-s^2} \\ &\quad |x| \leq 1-m \end{aligned}$$

Thus we get the following expressions for $D(r, t_0, l_0)$

$$\text{since } \int \cos^{-1}\left(\frac{r+\rho}{2\rho}\right) d\rho = \rho \cos^{-1}\left(\frac{r+\rho}{2\rho}\right) - \frac{1}{\sqrt{3}} \log \left\{ \sqrt{4\rho^2 - (r+\rho)^2} + \frac{3\rho-r}{\sqrt{3}} \right\}$$

$$\begin{aligned} D(r, t_0, l_0) &= \rho_0 \left[\cos^{-1} \frac{1+x}{2} - x \cos^{-1} x \right] + \frac{1}{\sqrt{3}} \log \left\{ \frac{\sqrt{4 - (1+x)^2}}{2x} + \frac{3-x}{2x} \right\} \\ &\quad \text{for } |x| > \sqrt{1-s^2} \quad |x| > 1 - \frac{s}{\rho} = 1-m \end{aligned}$$

$$\begin{aligned} &= \rho_0 \left[\cos^{-1} \left(\frac{1+x}{2} \right) - s \cos^{-1} \left(\frac{s+x}{2} \right) \right] + \frac{1}{\sqrt{3}} \log \left\{ \frac{\sqrt{4 - (1+x)^2} + (3-x)}{\sqrt{4s^2 - (x+s)^2} + (3s-x)} \right\} \\ &\quad \text{for } |x| \leq \sqrt{1-s^2}, \quad |x| > 1-m \end{aligned}$$

$$D = \rho_0 \left[\cos^{-1} \left(\frac{1+x}{2} \right) - s \cos^{-1} \left(\frac{s+x}{2} \right) \right] + \frac{1}{\sqrt{3}} \log \left\{ \frac{\sqrt{4-(1+x)^2} + 3-x}{\sqrt{4s^2-(s+x)^2} + 3s-x} \right\} -$$

$$- \rho_0 \left[\cos^{-1} \left(\frac{1+x+m}{2} \right) - s \cos^{-1} \left(\frac{s+x+m}{2} \right) \right] + \frac{1}{\sqrt{3}} \log \left\{ \frac{\sqrt{4-(1+x+m)^2} + 3-x-m}{\sqrt{4s^2-(s+x+m)^2} + 3s-x-m} \right\}$$

for $x \leq \sqrt{1-s^2}$, $x \leq 1-m$

In the curves given in Fig. 17 we have not used these analytic expressions for $D(rt_0, \ell, 0)$ but instead have performed the integrals (3) graphically.

In our spectrograph we have no photographic plate but instead have a slit. The distribution that we obtain will then be a sort of microphotometer curve of the photographic plate. i.e. we must pass a slit over the distribution given by formula (4). This integration we have also done graphically, determining the area of the density curve under the slit for various positions of the slit. We have taken the points by moving the slit 0.1 percent of ρ at a time.

The scale of the distribution of course varies with ρ getting narrower for smaller ρ . Our procedure of passing the slit over the distribution is thus not completely justified since it neglects the contracting of the line as ρ varies from the outside to the inside of the slit.

In our case the contraction is only 1 percent of the line width. As the line width is 2 percent of ρ , the contraction is negligible.

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